Visual Feedback Control of Planar Manipulators Based on Nonlinear Receding Horizon Control Approach

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Abstract

This paper deals with the vision-based robot motion control via the nonlinear receding horizon control approach. The visual feedback system consists of the manipulator dynamics and the image dynamics which is derived from the camera model. We propose the stabilizing receding horizon control scheme which is based on a control Lyapunov function and a corresponding feedback control law. The control Lyapunov function is constructed by the full Lagrangian dynamics based on the image feature parameter potential. The proposed scheme employs the cost function as a Lyapunov function for establishing stability of nonlinear receding horizon control. The effectiveness of the proposed scheme is illustrated by applying this approach to the planar model of visual feedback system.

1 Introduction

Vision based control of robotic systems involves the fusion of robot kinematics, dynamics, and computer vision systems to control the position of the robot end-effector. The combination of mechanical control with visual information should become extremely important, when we consider the mechanical systems working with targets whose position is unknown. Non-contact sensing is useful in the achievement of many kinds of the robotics tasks. Recent research efforts toward this direction have been nicely collected in [1] and [2].

This paper deals with the eye-in-hand approach to the visual feedback control. The typical example is shown in Fig. 1. This control problem is important, and has gained much attention of researchers for recent years [1]. However, much of previous works assume that the manipulator dynamics do not interact with the visual feedback loop. Although this assumption is valid for slow robot motion, it does not hold for high-speed tasks where the manipulator dynamics is not neglectable. Hence, it is important to deal with the visual feedback control problems in terms of the nonlinear dynamical control aspects [1], [3]. In particular, the Lyapunov function for the visual feedback systems has been shown explicitly in [4]–[11].

While, recently there has been a rapidly growing interest in using receding horizon control schemes for control of the nonlinear systems [12]. In [13], Ohtsuka et al. have applied the nonlinear receding horizon control to obstacle avoidance of a space-vehicle model. This interest is partly due to the availability of faster and cheaper computers as efficient numerical algorithms for solving optimization problems. In the area of the chemical process control, receding horizon control methods have been widely successful. This is due to the fact that many important industrial chemical processes are open-loop stable so that stability is not a primary concern for these methods. Several researchers have suggested different methods to guarantee the closed-loop stability of the receding horizon scheme. In a recent paper by De Nicolao et al. [14], the receding horizon controller guarantees closed-loop stability by using a possibly non quadratic end point penalty which is the cost incurred if a locally stabilizing linear control law is applied at the end of the time horizon T. Jadamba et al. [15] have followed the method of De Nicolao et al. by using the control Lyapunov function as the end point penalty, and have shown that stability of the receding horizon scheme is guaranteed.

In this paper, we propose the nonlinear receding horizon control scheme for the visual feedback system. The stability of the visual feedback system is discussed with the manipulator dynamics. A control Lyapunov function and a corresponding control law which have been proposed by authors in [10] play a crucial role for the proposed scheme.

Figure 1: Planar Visual Feedback Configuration
The paper is organized as follows. Section 2 shows the model of the visual feedback system. Section 3 introduces the control Lyapunov function and the corresponding visual feedback control law which are important for the receding horizon control. In Section 4, we propose the stabilizing receding horizon control scheme for the visual feedback system. Finally the numerical example and the conclusion are shown in Section 5 and 6, respectively.

2 System Model

The manipulator model considered here is the well-known Euler-Lagrange system whose inputs are joint torques and whose measurement outputs are joint positions and velocities. A pinhole camera, mounted on the hand of the manipulator in Fig. 1, is modeled by an ideal perspective transformation.

2.1 Manipulator Model

The dynamics of a n-link rigid manipulator can be expressed as

\[ M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau, \tag{1} \]

where

- \( q \): n \times 1 vector of the joint angles
- \( \tau \): n \times 1 vector of control input torques
- \( M(q) \): n \times n inertia matrix of the manipulator
- \( C(q, \dot{q})\dot{q} \): n \times 1 vector of the Coriolis and centrifugal torques
- \( g(q) \): n \times 1 vector of the gravitational torques.

The following properties are well known [16].

**Property 1** The inertia matrix \( M(q) \) is positive definite.

**Property 2** \( M(q) - 2C(q, \dot{q}) \) is skew-symmetric.

Property 2 is concerned with the passivity property. These properties are important for the Lyapunov/passivity based control design.

By using the inertia parameters, the dynamic equation (1) can be transformed as

\[ M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = Y(q, \dot{q}, \ddot{q})\theta = \tau, \tag{2} \]

where \( Y(q, \dot{q}, \ddot{q}) \) is an n \times p matrix of known functions, called the regressor, and \( \theta \) is a constant p-dimensional vector of the inertia parameters. The dimension of the parameter space is not unique.

2.2 Camera Model

We consider a planar manipulator with the world frame \( \Sigma_w = \{X_w, Y_w, Z_w\} \). It is assumed that the manipulator end-effector evolves in the \( X_w-Y_w \) plane of \( \Sigma_w \). Suppose that a camera with the frame \( \Sigma_c = \{X_c, Y_c, Z_c\} \) is mounted on the manipulator end-effector as depicted in Fig. 1. Hence, the manipulator kinematics gives the camera position \( \dot{w}_c(q) := [x_c(q) \ y_c(q)]^T \) and the orientation \( \dot{w}_c(q) \) with respect to \( \Sigma_w \). A frame \( \Sigma_i = \{X_i Y_i\} \) is defined in the camera image plane and its origin is the intersection of the optical axis with the image plane. Here it is assumed that the axes \( X_c \) and \( Y_c \) parallel the axes \( X_w \) and \( Y_w \), respectively, and the planes \( X_c - Y_c \) and \( X_w - Y_w \) are separated by the focal length \( i > 0 \).

Next the object point \( \dot{w}_p_o \) is located at \( [x_o \ y_o \ z_o] \) with respect to the frame \( \Sigma_w \). We assume that the object is static, i.e. \( \dot{w}_p_o = 0 \). \( \dot{w}_p_o = [x_o \ y_o] \) is the image coordinate of \( \dot{w}_p_o \) through the perspective transformation with the frame \( \Sigma_i \).

Taking the perspective transformation as the camera model (shown in Fig. 1) yields [6]-[10]

\[ \dot{t}_p := f(q) = \frac{s\lambda}{w_{zo}} R^t (\dot{w}_c(q)) (\dot{w}_p_o - \dot{w}_c(q)), \tag{3} \]

where \( s > 0 \) is the scaling factor in pixels/m, \( \dot{w}_p_o := [x_o \ y_o] \) and \( f: \mathbb{R}^2 \times \mathbb{R}^2 \rightarrow \mathbb{R}^2 \).

Then the differential kinematics of the manipulator gives the relationship between the manipulator joint velocities \( \dot{q} \) and the velocities of the camera mounted on the end-effector. The relation can be represented using the manipulator Jacobian \( J_p(q) \in \mathbb{R}^n \times n \):

\[ \dot{w}_p_c(q) = J_p(q)\dot{q}. \tag{4} \]

The derivation of the equation (3) yields

\[ f = -\frac{s\lambda}{w_{zo}} R^t J_p \dot{q} - R^t R f. \tag{5} \]

Now, we introduce Property 3 that is important for the Lyapunov/passivity based control design.

**Property 3** [16] \( R^t \dot{R} \) is skew-symmetric.

The following assumptions will be made throughout the paper:

**Assumption 1** There exists a manipulator joint configuration achieving \( f = 0 \).

**Assumption 2** The Jacobian \( J_p \) is nonsingular.

Assumption 1 ensures that the control problem is solvable. Assumption 2 is required for technical reasons in the stability analysis.

3 Visual Feedback Control

In this section, we discuss the stability of the visual feedback control. Since the visual feedback system consists of the equations (1) and (5) which are obtained in
Section 2, we consider the visual feedback system model described as
\[
M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau \quad (6)
\]
\[
f = -\frac{s\lambda}{w \omega_0} R'Jp\dot{p} - R'\dot{R}f. \quad (7)
\]
By invoking the discussions in [10]
\[
\tau = u + a M(q)\dot{q} + a C(q, \dot{q})\dot{q} + g(q)
= u + Y(q, \dot{q}, \alpha, \dot{\alpha}, \alpha \dot{\alpha})\theta, \quad (8)
\]
where $\eta := J_p Rf$ and $\alpha \in \mathbb{R}_+$. $u$ is the control input that will be designed to achieve the visual feedback control objective.

Substituting the control law (8) into (6) gives us the following temporally closed-loop system
\[
\dot{\xi} = -M^{-1}C\xi + M^{-1}u
\]
\[
f = \frac{s\lambda}{w \omega_0} R'Jp J_p' \dot{R}f - \frac{s\lambda}{w \omega_0} R'Jp \dot{\xi} - R'\dot{R}f, \quad (9)
\]
where $\xi := \dot{q} - \alpha J_p Rf$.

The following lemma plays a key role in the stability analysis performed below.

**Lemma 1** [10] Under the Assumption 1 and 2, consider the closed-loop system (9) with the following control law
\[
u = -K_1\xi + J_p Rf, \quad (10)
\]
where $K_1 \in \mathbb{R}^{n \times n}$ is positive definite. Then the equilibrium point $[\dot{\xi} \ f]' = 0$ of the system (9) is asymptotically stable. Furthermore the solutions of the closed-loop system (9) asymptotically converge to zero, i.e., $[\dot{\xi} \ f]' \to 0$, as $t \to \infty$.

**Proof:** By using Property 1, i.e. $M(q) > 0$, we can consider the positive definite function (11) as a Lyapunov function candidate
\[
V = \frac{1}{2} \xi' M(q) \xi + \frac{w \omega_0}{2s \lambda} \|f\|^2. \quad (11)
\]
The above Lyapunov function candidate is constructed by the full Lagrangian dynamics based on a potential function of the image feature parameter space, called image feature parameter potential which has been proposed by the authors in [6]. Evaluating the time derivative of $V$ along the trajectories to the system (9) gives us
\[
\dot{V} = \xi' M\dot{\xi} + \frac{1}{2} \xi' \dot{M} \xi + \frac{w \omega_0}{s \lambda} \xi' \dot{f} f
= \xi' M(-M^{-1}C\xi + M^{-1}u) + \frac{1}{2} \xi' \dot{M} \xi \\
+ \frac{w \omega_0}{s \lambda} \xi' \left( -\frac{s\lambda}{w \omega_0} R'Jp J_p' \dot{R}f - \frac{s\lambda}{w \omega_0} R'Jp \dot{\xi} - R'\dot{R}f \right).
\]
By using Property 2 and 3, i.e., $M = 2C$ and $R'\dot{R}$ are skew-symmetric, $V$ is transformed as
\[
\dot{V} = \frac{1}{2} \xi' (M - 2C) \xi + \xi' u - \alpha \int R' J_p J_p' R f f' \\
- \int R' J_p \dot{\xi} - \frac{w \omega_0}{s \lambda} \xi' \dot{R} f f
= \xi' u - \alpha \int R' J_p J_p' R f f' - \int R' J_p \dot{\xi}.
\]
Substituting the control law (10) into (12) yields
\[
\dot{V} = -\xi' K_1 \xi + \xi' J_p Rf - \alpha \int R' J_p J_p' R f f' - \int R' J_p \dot{\xi}
= -\xi' K_1 \xi - \alpha \int R' J_p J_p' R f f', \quad (13)
\]
which is the negative definite function for all $[\xi' f'] \neq 0$, since $R$ and $J_p$ are nonsingular. Hence the asymptotic stability can be confirmed. Further, $[\xi' f']' \to 0$ is equivalent to $[\xi' f']' \to 0$.

**Lemma 2** [10] The positive definite function (11) is a control Lyapunov function.

**Proof:** The equation (12) shows that
\[
\inf_u \{ \dot{V} \} = \begin{cases} 
\alpha \int R' J_p J_p' R f f' - \int R' J_p \dot{\xi} \xi' u & \text{if } \xi = 0 \\
-\infty & \text{if } \xi \neq 0
\end{cases}. \quad (14)
\]
Hence, the positive definite function (11) is a control Lyapunov function for the visual feedback system (9).

The above discussions give us the control Lyapunov function and the corresponding feedback control law for the visual feedback system. These are important for the nonlinear receding horizon control.

4 Nonlinear Receding Horizon Control

4.1 Review of Nonlinear Receding Horizon Control

Here we would like to review the nonlinear receding horizon control scheme which has been proposed in [15]. Consider the following Finite Horizon Optimal Control Problem (FHOCP).
\[
J(t, x, T, u) = \inf_{u \in U_{[t, t+T]}} \int_{t}^{t+T} h(x(\tau), u(\tau)) d\tau + M(x(t + T)), \quad (15)
\]
subject to $\dot{x} = f(x, u)$

where $h$ is a positive definite function of $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$. Now let $J^*(t, x, T, u)$ be the optimal value of the cost function of the FHOCP. At time $t$, FHOCP is solved over $[t, t+T]$ and the corresponding optimal control $u^*(\tau), t \leq \tau < t + T$ is computed. Then, the current control is set equal to $u^*(t)$. Repeating these calculations yields a feedback control law. To ensure closed-loop stability, the following results was proved by Jadabaie et al. [15].
Lemma 3  The receding horizon optimal control scheme (16) is asymptotically stabilizing if a stabilizing feedback control law \( u_k(x) \) is available.

\[
M(x(t + T)) = \int_{t}^{\infty} h\left( \phi_2(\tau; t + T, x^*_\tau, u_k), \right) \\
u_k(\phi_2, (t + T, x^*_\tau, u_k)) \, d\tau
\]

\[ x^*_\tau = \phi_1(t, x(t), T, u^*(t, x, T)) \]  

\[ u^*(t, x, T) = \arg\inf_u J(t, x, T, u), \]

where \( \phi_1 \) is the flow of the vector field along the open-loop receding horizon trajectory \( u^* \), and \( \phi_2 \) is the flow along the feedback control law \( u_k \) obtained in advance.

Lemma 4  Consider the FHOCP with the following terminal penalty:

\[
M(x(t + T)) = \rho V(x(t + T)),
\]

where \( V \) is a control Lyapunov function obtained a priori and \( \rho \) is a design parameter. Denote the optimal cost associated with this problem by \( J^*_T \). Then there exists a \( \rho_0 \) such that for all \( \rho \geq \rho_0, \) \( J^*_T \) is a Lyapunov function for the closed-loop system with the receding horizon feedback.

4.2 Nonlinear Receding Horizon Control with Visual Feedback System

In this subsection, we consider the stability of the visual feedback system via the nonlinear receding horizon control approach.

Consider the Finite Horizon Optimal Control Problem (FHOCP) for the visual feedback system (9) which is based on the following optimization

\[
J(t, x, T, u) = \inf_{u} \int_{t}^{t + T} h(\xi(t), f(\tau), u(\tau)) \, d\tau \\
+ \rho V(x(t + T)).
\]

Let the cost \( h(\xi, f, u) \) be as follows

\[
h(\xi, f, u) = \xi^T Q_1 \xi + f^T R^T J_p Q_2 f + u^T \mathcal{R} u,
\]

where \( Q_1 > 0, Q_2 > 0, \) and \( \mathcal{R} > 0. \) \( \rho \) is a design parameter and \( V \) is a terminal penalty.

To ensure closed-loop stability, we consider the following receding horizon optimal control scheme.

\[
V(x(t + T)) = \frac{1}{2} \xi^T(t + T)M \xi(t + T) \\
+ \frac{w}{2n} \xi(t + T) \eta(t + T) \]

\[ x^*(t + T) = \phi_1(t, x(t), T, u^*(t, x, T)) \]  

\[ u^*(t, x, T) = \arg\inf_u J(t, x, T, u). \]

where \( x := [\xi, f]^T \) and \( V \) is a control Lyapunov function which has been proved in Lemma 2. At time \( t \), the finite horizon optimal control problem is solved over \([t, t + T]\) and the corresponding optimal control law \( u^*(\tau), t \leq \tau < t + T \) is computed (shown in Fig. 2). Then, the optimal control trajectory is set equal to \( u^*(t, x, T) \) and the current optimal control law is defined as \( u^*(t) \). At the next time instant, the whole procedure is repeated. \( \phi_1 \) is the flow of the vector field along the open-loop receding horizon trajectory \( u^* \).

\[
Figure 2: \text{Receding horizon approach}
\]

Using the argument presented in Lemma 1–4 we have the following theorem.

Theorem 1  Consider the FHOCP (9) and (18) with the following control law

\[
u_k = -K_1 \xi + J_p f,
\]

where \( \xi := \dot{q} - \alpha \eta \) and \( K_1 \in \mathbb{R}^{n \times n} \) is positive definite. Then the receding horizon optimal control scheme (20)–(22) is asymptotically stabilizing.

Remark 1  \( u_k \) is a stabilizing control law for the visual feedback system which has been proved in Lemma 1. An important note is that the stabilizing control law \( u_k \) is never actually applied, but it is just used to compute the end point penalty.

Proof: Our goal is to prove that \( J^* (t, x, T, u) \) will qualify as a Lyapunov function for the closed loop system. We construct the following sub-optimal strategy for the time interval \([t, t + T + \delta]\]

\[
u = \begin{cases} 
    u^*(\tau), & \tau \in [t, t + T] \\
    u_k(\tau), & \tau \in [t + T, t + T + \delta]
\end{cases}
\]

where \( u_k \) is a stabilizing feedback control law for the closed-loop system (9). The stabilizing feedback control law (23) was proposed by the authors in [10].

\( J^* (t, x, T, u) \) is positive definite, and the time derivative
of $J^*(t, x, T, u)$ is as follows.

$$J^*(t, x, T, u) = \rho \mathcal{V}(x_T) + \xi_T^T \mathbf{Q}_1 \xi_T + \int_{t}^{T} \mathbf{f}_R R^T \mathbf{Q}_R \mathbf{J}_R \mathbf{f}_R dt + \mathbf{u}_k R \mathbf{u}_k - \left( \xi_T \mathbf{Q}_1 \xi_T + f^T \mathbf{R} \mathbf{J}_Q \mathbf{J}_R^T \mathbf{R} f + u^T \mathbf{R} \mathbf{u} \right).$$

By using Property 2 and 3, $\mathcal{V}(x_T)$ is transformed as

$$\mathcal{V}(x_T) = \frac{1}{2} \xi_T^T \left( \mathbf{M} - 2C \right) \xi_T + \xi_T^T \mathbf{u}_k$$

Substituting the stabilizing feedback control law (23) into (26), we obtain

$$\dot{\mathcal{V}}(x_T) = \xi_T^T \left( -K_1 \xi_T + J_{\mu_T}^T R_{\mu_T} f_T \right)$$

Hence, the equation (25) and the control law (23) give us

$${J}^*(t, x, T, u) = \rho \mathcal{V}(x_T) + \xi_T^T \mathbf{Q}_1 \xi_T + \int_{t}^{T} \mathbf{f}_R R^T \mathbf{Q}_R \mathbf{J}_R \mathbf{f}_R dt + \mathbf{u}_k R \mathbf{u}_k$$

Moreover, $\dot{J}^*(t, x, T, u)$ can be formulated as

$$\dot{J}^*(t, x, T, u) = -x_T^T \mathbf{P} x_T - \xi_T^T \mathbf{Q}_1 \xi_T - \mathbf{f}^T \mathbf{R} \mathbf{J}_Q \mathbf{J}_R^T \mathbf{R} f - u^T \mathbf{R} \mathbf{u}.$$  (28)

We discussed the stabilizing receding horizon control scheme for the visual feedback system. Our proposed scheme is based on the control Lyapunov function and the corresponding feedback control law.

5 Numerical Example

To illustrate the behavior of the visual feedback control, we apply the receding horizon control to a two-link planar manipulator with an eye-in-hand system. The entries of the inertia matrix $\mathbf{M}(q)$ and the Coriolis and centrifugal matrix $C(q, \dot{q})$ are given by

$$\mathbf{M}(q) = 
\begin{bmatrix}
M_1 + M_2 + 2R_1 \cos q_2 & M_2 + R_1 \cos q_2 \\
M_2 + R_1 \cos q_2 & M_2
\end{bmatrix}
$$

$$C(q, \dot{q}) = 
\begin{bmatrix}
-R_1 \dot{q}_1 \sin q_2 & -R_1 (\dot{q}_1 + \dot{q}_2) \sin q_2 \\
R_1 \dot{q}_1 \sin q_2 & 0
\end{bmatrix}
$$

Since the robot moves in the horizontal plane, we have $g(q) = 0 \in \mathbb{R}^3$. The rotation matrix $R$ and the Jacobian matrix $J_p$ are described as

$$R = \begin{bmatrix}
\cos(q_1 + q_2) & -\sin(q_1 + q_2) \\
\sin(q_1 + q_2) & \cos(q_1 + q_2)
\end{bmatrix}
$$

$$J_p = \begin{bmatrix}
-l_1 \sin q_1 - l_2 \sin(q_1 + q_2) & -l_2 \sin(q_1 + q_2) \\
l_1 \cos q_1 + l_2 \cos(q_1 + q_2) & l_2 \cos(q_1 + q_2)
\end{bmatrix}
$$

The distance between the optical center and robot workspace plane is $w_z = 1.86$ m. It is assumed that the focal length $\lambda$ multiplied by the scale factor $s$, i.e. $s \lambda$, is equal to 2180 pixels. The target has been placed in the $X_w = Y_w$ plane at $w_p = \left[ 
\begin{array}{c}
0.3414 \\
-0.1414 \end{array}
\right]$. The initial conditions of the robot positions and velocities are set as follows: $x(0) = 0$ rad, $\dot{x}(0) = \pi/2$ rad, and $\dot{\xi}(0) = \dot{\theta}_2(0) = 0$ rad/s. Hence, the initial state is $x(0) = \left[ \begin{array}{c} \xi_1 \\ \xi_2 \\ f_1 \\ f_2 \end{array} \right] = \left[ 
\begin{array}{c}
113 \\
33 \\
-400 \\
-165 \end{array}
\right]$. The gain matrices are chosen as $\mathbf{Q}_1 = \text{diag}(4, 2)$, $\mathbf{Q}_2 = \text{diag}(4, 2)$, and $\mathbf{R}_1 = \text{diag}(4, 2)$. We pick $\rho = 10$.
and the horizon length $T = 0.05$ with a sampling time 0.01 s.

In receding horizon control, the current control at state $x$ and time $t$ is obtained by determining on-line (the open-loop) optimal control $u^*(t, x, T)$ over the interval $[t, t + T]$ and setting the control equal to $u^*(t)$. Repeating this calculation continuously yields a feedback control (since $u^*(t)$ clearly depends on the current state $x$). The optimal control problem (9) and (18) is solved on-line. Fig. 3 shows the control input $u$, and Fig. 4 shows the image position error $f$ and $\xi$ tend asymptotically to zero. It can be seen that the equilibrium point of the visual feedback system is asymptotically stable.

6 Conclusion

In this paper, the nonlinear receding horizon control for the visual feedback system has been discussed. In particular, we proposed the stabilizing receding horizon control scheme which is based on the control Lyapunov function and the corresponding feedback control law. The proposed scheme has employed the cost function as a Lyapunov function for establishing stability of nonlinear receding horizon control. Moreover, the numerical example was reported to illustrate the effectiveness of the proposed scheme.

References


