A Design Scheme of Passivity-based Position and Force Control for Mobile Manipulator

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Abstract: This paper considers the passivity-based position and force control for a mobile manipulator. It is assumed that the mobile manipulator is composed of the wheeled mobile vehicle and 2-link manipulator. The kinematics and dynamics model of the wheeled mobile vehicle and 2-link manipulator is considered. The passivity-based control is applied to the wheeled mobile vehicle and 2-link manipulator. It is shown that the stability of the mobile manipulator is guaranteed by using a Lyapunov function. In the simulation, it is indicated that the proposed control law can track the reference trajectories for the unknown external force.

Keywords: passivity-based control, mobile manipulator, force control

1. INTRODUCTION

Recently, it is expected that mobile manipulators are used for rescue work of the natural disaster instead of workers. The mobile manipulator consists of the wheeled mobile vehicles and manipulators. It is required that the wheeled mobile vehicle and the manipulator track the reference trajectory by the force control, because the dynamics of the mobile manipulator should be considered. The mobile manipulators have been focus in control problems [1, 2].

In previous research [1, 2], the hybrid position and force control is considered for the mobile manipulator. The adaptive control is applied to the mobile manipulator with nonholonomic constraints. It is shown that the mobile manipulator can converge to the reference trajectories of the position and force.

This paper considers the passivity-based position and force control for a mobile manipulator. It is assumed that the mobile manipulator is composed of the wheeled mobile vehicle and 2-link manipulator. The kinematics and dynamics model of the wheeled mobile vehicle and 2-link manipulator is considered. The passivity-based control is applied to the wheeled mobile vehicle and 2-link manipulator. Though, it is difficult to track the nonholonomic constraints, the passivity-based control can utilize the property of the mobile manipulator dynamics. The aim of our research is to propose the control law that the mobile manipulator track the reference trajectories.

First, the kinematics and dynamics model of the mobile manipulator is indicated. It is shown that the stability of the mobile manipulator is guaranteed by using a Lyapunov function. For the external force, the constraints of design parameters are indicated by using the L2 gain. It is indicated that the proposed control law can track the reference trajectories for the unknown external force. In the simulation, the performance of the trajectory tracking is verified by utilizing MATLAB. It is shown that the mobile manipulator can track the reference trajectories which are composed of the straight line and curve line.

2. PROBLEM FORMULATION

Consider the mobile manipulator which is composed of the wheeled mobile vehicle and 2-link manipulator as shown in Fig. 1. The coordinate of the wheeled mobile vehicle consists of the position $(x_B, y_B)$ and the rotation $\theta_B$. The angle velocities of right and left wheels is defined as $\theta_{Br}, \theta_{Bl}$, respectively. The generalized coordinate $q_B$ is written as

$$q_B = [x_B, y_B, \theta_B, \theta_{Br}, \theta_{Bl}]^T. \quad (1)$$

The joint angles of the 2-link manipulator are $q_{M1}, q_{M2}$. The state vectors of the vehicle and manipulator becomes

$$q_M = [q_{M1}, q_{M2}]^T. \quad (2)$$

The generalized coordinate is represented as

$$q = [q_B^T, q_M^T]^T. \quad (3)$$

![Fig. 1 Coordinate of mobile manipulator](image)

The torques of the mobile manipulator consists of $\tau_{Br}, \tau_{Bl}, \tau_{M1}, \tau_{M2}$, where $\tau_{Br}, \tau_{Bl}$ are the wheeled mobile vehicle’s torque, $\tau_{M1}, \tau_{M2}$ are 2-link manipulator’s torque. The symbol $r_B, b_B$ is the radius of the wheels and the half-width of the wheeled vehicle, respectively. The $d$ is distance from the center of the shaft of wheels to the coordinate origin of 2-link manipulator.

The 2-link manipulator is shown in Fig. 2. The $l_1, l_2$ are length of 2-link manipulator, $r_1, r_2$ are length
of the center of gravity. The $m_B, m_W$ are the mass of body of wheeled mobile vehicle and wheels. The $m_1, m_2$ are the mass of the 2-link manipulator, The $I_B, I_W, I_m, I_{M1}, I_{M2}$ are inertia of wheeled mobile vehicle, shaft of wheels, wheels, 2-link manipulator.

![Fig. 2 Coordinate of 2-link manipulator](image)

The kinematics model of wheeled vehicle is shown as

$$
\begin{bmatrix}
\frac{dx_B}{dt} \\
\frac{dy_B}{dt} \\
\frac{d\theta_B}{dt} \\
\frac{d\theta_{BI}}{dt}
\end{bmatrix} =
\begin{bmatrix}
\cos \theta_B & 0 & 0 \\
\sin \theta_B & 0 & 1 \\
\frac{1}{r_B} & \frac{2}{r_B} & -\frac{2}{r_B}
\end{bmatrix}
\begin{bmatrix}
u_B \\
\omega_B
\end{bmatrix}
\tag{4}
$$

where $u_B$ is the velocity of wheeled vehicle, $\omega_B$ the angle velocity. The $u_B, \omega_B$ are related to the angle velocities of the right and left wheels $\nu_B = [\dot{\theta}_{Br}, \dot{\theta}_{Bl}]^T$

$$
\begin{bmatrix}
u_B \\
\omega_B
\end{bmatrix} =
\begin{bmatrix}
\frac{\nu_B}{2r_B} \\
\frac{-\nu_B}{2r_B}
\end{bmatrix}
\begin{bmatrix}
\dot{\theta}_{Br} \\
\dot{\theta}_{Bl}
\end{bmatrix}
\tag{5}
$$

Thus, the kinematics model of (4) can be represented

$$
\begin{bmatrix}
\frac{dx_B}{dt} \\
\frac{dy_B}{dt} \\
\frac{d\theta_B}{dt} \\
\frac{d\theta_{BI}}{dt}
\end{bmatrix} =
\begin{bmatrix}
\frac{\nu_B}{2r_B} & \frac{\nu_B}{2r_B} & -\frac{\nu_B}{2r_B} & 0 \\
\frac{\nu_B}{2r_B} & \frac{\nu_B}{2r_B} & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\dot{\theta}_{Br} \\
\dot{\theta}_{Bl}
\end{bmatrix}
\Rightarrow \dot{q}_B = S_B(q_B)\nu_B.
\tag{6}
$$

By using the equation (6), the kinematics model of mobile manipulator can be given as follows

$$
\dot{q} = S(q_B)\nu
\tag{7}
$$

where

$$
S(q_B) = \begin{bmatrix}
S_B(q_B) & 0 \\
0 & I_2
\end{bmatrix}, \quad \nu = \begin{bmatrix}
\nu_B \\
\dot{q}_M
\end{bmatrix}.
\tag{8}
$$

The $S(q_B)$ is matrix which represents between $\nu$ and generalized coordinate $q$. The $\nu$ is composed of the angle velocity of wheeled vehicle $\nu_B$ and 2-link manipulator $\dot{q}_M$.

From Fig. 2, the coordinate of hand is represented as

$$
x_M = l_1 \cos q_{M1} + l_2 \cos(q_{M1} + q_{M2})
$$

$$
z_M = l_1 \sin q_{M1} + l_2 \sin(q_{M1} + q_{M2})
\tag{9}
$$

The Jacobian of manipulator $J_M(q)$ is defined as

$$
J_M(q) = [J_{M0} \quad J_{M2}]
\tag{10}
$$

where

$$
\frac{\partial}{\partial \dot{q}_B} \begin{bmatrix}
x_M \\
z_M
\end{bmatrix} = J_{M0}
\tag{11}
$$

$$
\frac{\partial}{\partial \dot{q}_M} \begin{bmatrix}
x_M \\
z_M
\end{bmatrix} = J_{M2}
\tag{12}
$$

Therefore, Jacobian $J_M(q)$ is derived

$$
J_M(q) = \begin{bmatrix}
0 & 0 & 0 & 0 & -l_1 S_1 - l_2 S_{12} \\
0 & 0 & 0 & 0 & l_1 C_1 + l_2 C_{12}
\end{bmatrix}
\tag{13}
$$

where

$$
S_1 := \sin q_{M1}, \quad C_1 := \cos q_{M1}, \quad S_{12} := \sin(q_{M1} + q_{M2}), \\
C_{12} := \cos(q_{M1} + q_{M2}).
$$

The dynamics of mobile manipulator is given by the dynamics of wheeled mobile vehicle and 2-link manipulator

$$
M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + G(q) = J_M^T(q) \lambda_M + B(q)\tau
\tag{14}
$$

where $M(q) \in \mathbb{R}^{7 \times 7}$ is the positive definite inertia matrix, $C(q, \dot{q}) \in \mathbb{R}^{7 \times 7}$ represents the Coriolis and centrifugal torques, $G(q) \in \mathbb{R}^7$ is the gravitational torques, $J_M(q) \in \mathbb{R}^{7 \times 7}$ is the Jacobian and $B(q)\tau \in \mathbb{R}^7$ is the control input and $\lambda_M \in \mathbb{R}^7$ is external force. The component of matrices are written as

$$
M(q) = \begin{bmatrix}
M_B & 0 \\
0 & M_M
\end{bmatrix}, \quad C(q, \dot{q}) = \begin{bmatrix}
C_B & 0 \\
0 & C_M
\end{bmatrix},
$$

$$
G(q) = \begin{bmatrix}
0 \\
G_M
\end{bmatrix}, \quad B(q) = \begin{bmatrix}
B_B & 0 \\
I_2 & I_2
\end{bmatrix}, \quad \lambda_M = \begin{bmatrix}
\lambda_{M1} \\
\lambda_{M2}
\end{bmatrix},
$$

$$
\tau = \begin{bmatrix}
\tau_B \\
\tau_M
\end{bmatrix}.
$$

The state $q$ of the dynamics of mobile manipulator (14) is replaced as $q$ of kinematics model (7), the $S_M^T(q_B)$ is multiplied from left side, the dynamics model of mobile manipulator considering kinematics can be derived as follows

$$
M_1(q) \ddot{q} + C_1(q, \dot{q}) \dot{q} + G_1(q) = B_1(q)\tau + J_M^T(q)\lambda_M
\tag{15}
$$

$$
M_1(q) = S_M^T(q_B)M(q)S(q_B),
$$

$$
C_1(q, \dot{q}) = S_M^T(q_B)(M(q)\dot{S}(q_B) + C(q, \dot{q})S(q_B))
$$

$$
G_1(q) = S_M^T(q_B)G(q)
$$

$$
B_1(q) = S_M^T(q_B)B(q)
$$

$$
J_M^T = S_M^T(q_B)J_M^T(q)
$$

where $\dot{M}_1 - 2C_1$ is skew symmetric. The control objective is to track reference trajectories and to compensate external forces.
3. STABILITY OF MOBILE MANIPULATOR

When the external force \( \lambda_M = 0 \), the dynamics of the mobile manipulator is given by

\[
M_1(q) \ddot{q} + C_1(q, \dot{q}) \dot{q} + G_1(q) = B_1(q) \tau.
\]

(16)

The reference velocity of wheeled vehicle is defined as \( \eta_{Bd} = [\dot{\theta}_{Bd}, \omega_{Bd}]^T \), the reference angle velocity \( \nu_{Bd} = [\dot{\theta}_{Bd}, \dot{\theta}_{Bd}]^T \) is obtained from (5). Using the reference angle velocity of wheeled vehicle \( \nu_{Bd} \) and 2-link manipulator \( \dot{q}_{Md} = [\dot{q}_{M1d}, \dot{q}_{M2d}]^T \), the reference of mobile manipulator is redefined as

\[
\nu_d = [\dot{\theta}_{Bd} \dot{\theta}_{Bd} \dot{q}_{M1d} \dot{q}_{M2d}]^T.
\]

The error about the angle velocities of wheels and manipulator is defined

\[
\xi := \nu - \nu_d
\]

(17)

The input torque is given as follows

\[
B_1(q)\tau = M_1(q)\nu_d + C_1(q, \dot{q})\nu_d + G_1(q) + u_\xi
\]

(18)

where \( u_\xi \) is another input which is given after part.

By using the dynamics of mobile manipulator (16) and input torque (18), the whole system can be derived

\[
M_1(q)\dot{\xi} = -C_1(q, \dot{q})\xi + u_\xi.
\]

(19)

For the system (19), the control law which stabilizes the equilibrium point \( \xi = 0 \) is proposed as follows

\[
u_\xi = -K_\xi \xi
\]

(20)

\[
K_\xi := \begin{bmatrix}
k_{\xi 1} & 0 & 0 & 0 \\
0 & k_{\xi 2} & 0 & 0 \\
0 & 0 & k_{\xi 3} & 0 \\
0 & 0 & 0 & k_{\xi 4}
\end{bmatrix}
\]

(21)

where \( K_\xi \) is weight coefficient matrix for the wheels and links. Then, the stability of the equilibrium point \( \xi = 0 \) for the system (19) is considered.

**Theorem 1:** Consider the system (19), (20), the equilibrium point \( \xi = 0 \) of the system is asymptotically stable.

**Proof:** Define the energy function

\[
V_\xi = \frac{1}{2} \xi^T M_1(q) \xi.
\]

(22)

The derivative of above equation is given by

\[
\dot{V}_\xi = \frac{1}{2} \left( \dot{\xi}^T M_1(q) \xi + \xi^T \dot{M}_1(q) \xi + \xi^T M_1(q) \dot{\xi} \right)
\]

\[
= \xi^T M_1(q) \dot{\xi} + \frac{1}{2} \xi^T \dot{M}_1(q) \xi
\]

\[
= \xi^T (-C_1(q, \dot{q}) \xi + u_\xi) + \frac{1}{2} \xi^T \dot{M}_1(q) \xi
\]

\[
= \xi^T u_\xi + \frac{1}{2} \xi^T \left( \dot{M}_1(q) - 2C_1(q, \dot{q}) \right) \xi.
\]

(23)

Because the \( \dot{M}_1(q) - 2C_1(q, \dot{q}) \) is skew symmetric matrix

\[
\xi^T \left( \dot{M}_1(q) - 2C_1(q, \dot{q}) \right) \xi = 0
\]

(24)

the equation (23) is derived

\[
\dot{V}_\xi = \xi^T u_\xi = u_\xi^T \xi = -(K_\xi \xi)^T \xi = -\xi^T K_\xi \xi \leq 0.
\]

(25)

The stability of the equilibrium point \( \xi = 0 \) is guaranteed. The asymptotic stability of the equilibrium point \( \xi = 0 \) is proved by using LaSalle’s invariant principle, because \( \dot{V}_\xi \neq 0 \) except for \( \xi = 0 \).

4. EXTERNAL FORCE ATTENUATION

In case that the external force exists, the external force attenuation is considered. The system which is composed of (15), (18) is derives as follows

\[
M_1(q)\dot{\xi} = -C_1(q, \dot{q})\xi + u_\xi + J_{M}^T(q)\lambda_M.
\]

(26)

The equation (26) is substituted into (22)

\[
\dot{V}_\xi = \xi^T \left(-C_1(q, \dot{q})\xi + u_\xi + J_{M}^T(q)\lambda_M \right)
\]

\[
+ \frac{1}{2} \xi^T M_1(q) \xi
\]

\[
= \xi^T u_\xi + \xi^T J_{M}^T(q)\lambda_M
\]

(24)

\[
= \frac{\gamma^2}{2} ||\lambda_M||^2 - \frac{\gamma^2}{2} ||\lambda_M||^2 + \frac{1}{2} ||\xi||^2 - \frac{1}{2} ||\xi||^2
\]

\[
+ \xi^T u_\xi + \xi^T J_{M}^T(q)\lambda_M.
\]

(27)

Here, the following relation is satisfied

\[
\frac{\gamma^2}{2} ||\lambda_M - \frac{1}{\gamma^2} J_{M}(q)\xi ||^2
\]

\[
= \frac{\gamma^2}{2} \left( \lambda_M - \frac{1}{\gamma^2} J_{M}(q)\xi \right)^T \left( \lambda_M - \frac{1}{\gamma^2} J_{M}(q)\xi \right)
\]

\[
= \frac{\gamma^2}{2} ||\lambda_M||^2 - \frac{1}{2} \xi^T J_{M}^T(q)\lambda_M - \frac{1}{2} \lambda_M^2 J_{M}(q)\xi
\]

\[
+ \frac{1}{2} \gamma^2 \xi^T J_{M}^T(q) J_{M}(q)\xi
\]

\[
= \frac{\gamma^2}{2} ||\lambda_M||^2 - \xi^T J_{M}^T(q)\lambda_M + \frac{1}{2} \gamma^2 \xi^T J_{M}(q)\xi.
\]

(28)

Thus, the following equation is derived

\[
\xi^T J_{M}^T(q)\lambda_M = \frac{\gamma^2}{2} ||\lambda_M||^2 + \frac{1}{2} \gamma^2 \xi^T J_{M}(q)\xi
\]

\[
- \frac{\gamma^2}{2} ||\lambda_M - \frac{1}{\gamma^2} J_{M}(q)\xi ||^2
\]

(30)

where \( J = J_{M}^T(q) J_{M}(q) \). The equation (30) is substituted into (27)

\[
\dot{V}_\xi + \frac{1}{2} ||\xi||^2 - \frac{\gamma^2}{2} ||\lambda_M||^2
\]

\[
= -\frac{\gamma^2}{2} ||\lambda_M||^2 + \frac{1}{2} ||\xi||^2 + \xi^T u_\xi + \xi^T J_{M}^T(q)\lambda_M
\]

\[
= -\frac{\gamma^2}{2} ||\lambda_M||^2 + \frac{1}{2} ||\xi||^2 + \xi^T u_\xi + \frac{\gamma^2}{2} ||\lambda_M||^2
\]

\[
+ \frac{1}{2} \gamma^2 \xi^T J_{M}(q)\xi - \frac{\gamma^2}{2} ||\lambda_M - \frac{1}{\gamma^2} J_{M}(q)\xi ||^2
\]

\[
= \frac{1}{2} ||\xi||^2 - \xi^T K_\xi \xi + \frac{1}{2} \gamma^2 \xi^T J_{M}(q)\xi
\]

(31)
Then, if $\lambda_M$ is the worst disturbance
\[
\lambda_M = \frac{1}{\gamma^2} J_M(q) \xi
\] (32)
the following inequality consists for all $\lambda_M$.
\[
\dot{V}_\xi + \frac{1}{2} \| \xi \|^2 - \frac{\gamma^2}{2} \| \lambda_M \|^2 \\
\leq \frac{1}{2} \| \xi \|^2 - \xi^T K_\xi \xi + \frac{1}{2\gamma^2} \xi^T J \xi \\
= -\xi^T \left( K_\xi - \frac{1}{2\gamma^2} J - \frac{1}{2} I \right) \xi.
\] (33)
The matrix $P$ is defined
\[
P := K_\xi - \frac{1}{2\gamma^2} J - \frac{1}{2} I
\] (34)
where matrix $J$ is semi-positive definite matrix. If the matrix $P$ is semi-positive definite matrix
\[
\dot{V}_\xi + \frac{1}{2} \| \xi \|^2 - \frac{\gamma^2}{2} \| \lambda_M \|^2 \leq -\xi^T P \xi \leq 0
\] (35)
is satisfied. In order to satisfy that the matrix $P$ is semi-positive definite, it is necessary that equation (34) satisfies the following relation
\[
K_\xi \geq \frac{1}{2\gamma^2} J + \frac{1}{2} I.
\] (36)
Here, the constant matrix $J^*$ is proposed, where $J^*$ satisfies $J^* > J$. By using the matrix $J^*$, the following inequality is obtained
\[
K_\xi \geq \frac{1}{2\gamma^2} J^* + \frac{1}{2} I > \frac{1}{2\gamma^2} J + \frac{1}{2} I.
\] (37)
We proposes the matrix $J^*$
\[
J^* = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 3l_1^2 + 3l_2^2 + 2l_1 l_2 & 3l_2^2 + l_1^2 & 3l_2^2 + l_1^2 \\
0 & 3l_2^2 + l_1^2 & 3l_2^2 + l_1^2 & 3l_2^2 \\
\end{bmatrix}
\] (38)
From the inequality (37)
\[
K_\xi = \frac{1}{2\gamma^2} J^* - \frac{1}{2} I = \\
= \begin{bmatrix}
k_{\xi 1} + \frac{1}{4} \\
0 & k_{\xi 2} - \frac{1}{4} & 0 & 0 \\
0 & 0 & k_{\xi 3} + a & c \\
0 & 0 & c & k_{\xi 4} + b \\
\end{bmatrix}
\] (39)
where
\[
a = -\frac{1}{2\gamma^2} (3l_1^2 + 3l_2^2 + 2l_1 l_2) - \frac{1}{2}
\]
\[
b = -\frac{1}{2\gamma^2} 3l_2^2 - \frac{1}{2}
\]
\[
c = -\frac{1}{2\gamma^2} (3l_2^2 + l_1 l_2).
\]
The eigenvalue of matrix (39) is equal or more than 0, matrix $P$ becomes semi-definite matrix. By using Schur complement, the condition of gain $K_\xi$ that matrix $P$ is semi-positive definite matrix, is derives as follows.
\[
k_{\xi 1} - \frac{1}{2} \geq 0
\] (40)
\[
k_{\xi 2} - \frac{1}{2} \geq 0
\] (41)
\[
k_{\xi 3} - \frac{1}{2} \geq 0
\] (42)
\[
k_{\xi 4} - \frac{1}{2} \geq 0
\] (43)
From above equations, the following theorem is derived.

**Theorem 2:** Consider the system (19), (20), the design parameters $K_\xi$ is obtained following inequality (40)-(43). The $L_2$ gain from the external force $\lambda_M$ to $\xi$ becomes equal or less than $\gamma$
\[
\gamma \geq \sup_{\lambda_M \in L_2/\{0\}} \frac{\| \xi \|_{L_2}}{\| \lambda_M \|_{L_2}}
\] (44)

**Proof:** Choosing the $K_\xi$ which satisfy (40)-(43), the semi-positive definite matrix $P$ is obtained. Integrating the (35)
\[
\int_0^T (\gamma^2 \| \lambda_M \|^2 - \| \xi \|^2) dt \\
\geq 2 \int_0^T \dot{V}_\xi dt \geq -2V(0) := -\beta_{L_2}
\] (45)
where $\beta_{L_2}$ is non-negative constant which depends initial condition.
\[
\int_0^T \| \xi \|^2 dt \leq \gamma^2 \int_0^T \| \lambda_M \|^2 dt + 2V(0).
\] (46)
When $T \to \infty$, initial value is ignored
\[
\gamma \geq \sup_{\lambda_M \in L_2/\{0\}} \frac{\| \xi \|_{L_2}}{\| \lambda_M \|_{L_2}}
\] (47)

By using the condition $K_\xi$, the semi-positive definite matrix $P$ is defined, the system satisfy the $L_2$ gain from the external force $\lambda_M$ to controlled value $\xi$. Therefore, the controller which satisfies $\gamma \leq 1$ is designed, the external force $\lambda_M$ does not influence controlled value $\xi$. The mobile manipulator can track the reference trajectory without the influence of the external force.

**5. SIMULATION**

The model parameters are given as
\[
b_B = r_B = d = 1 \text{ [m]} \\
l_1 = l_2 = 2 \text{ [m]} \\
r_1 = r_2 = 1 \text{ [m]} \\
m_B = m_W = m_1 = m_2 = 1 \text{ [kg]} \\
I_B = I_W = I_m = I_1 = I_2 = 1 \text{ [kg · m²]}
The initial conditions are given
\[
\begin{align*}
\dot{q}(0) &= [0, 0, 0, 0, 0, 0]^T, \\
q(0) &= [0, 0, 0, 0, 0, 0]^T.
\end{align*}
\] (48)

The reference trajectory of wheeled mobile vehicle
\[
q_{Bd} = [x_{Bd}, y_{Bd}, \theta_{Bd}, \theta_{Bd, \gamma}]^T
\]
consists of the right line and the curve line. From (4), the reference velocity \(q\) is given as follows
\[
\begin{align*}
u_{Bd} &= 0.25(1 - \cos \frac{\pi}{5} t) \quad (0 \leq t < 5) \\
\omega_{Bd} &= 0 \\
u_{Bd} &= 0.5 \quad (5 \leq t < 10) \\
\omega_{Bd} &= 0 \\
u_{Bd} &= 0.25\{1 + \cos \frac{\pi}{5}(t - 10)\} \quad (10 \leq t < 15) \\
\omega_{Bd} &= 0 \\
u_{Bd} &= 0.15\pi\{1 - \cos \frac{\pi}{5}(t - 15)\} \quad (15 \leq t < 25) \\
\omega_{Bd} &= -(u_{Bd})/2.5 \\
u_{Bd} &= 0.15\pi\{1 - \cos \frac{\pi}{5}(t - 25)\} \quad (25 \leq t < 35) \\
\omega_{Bd} &= (u_{Bd})/2.5 \\
u_{Bd} &= 0.25\{1 - \cos \frac{\pi}{5}(t - 35)\} \quad (35 \leq t < 40) \\
\omega_{Bd} &= 0 \\
u_{Bd} &= 0.5 \quad (40 \leq t < 45) \\
\omega_{Bd} &= 0 \\
u_{Bd} &= 0.25(1 - \cos \frac{\pi}{5} t) \quad (45 \leq t < 50) \\
\omega_{Bd} &= 0
\end{align*}
\] (49)

When reference trajectory and of manipulator have uniformly-accelerated motion at the time of start and stop. The reference trajectories \(q_{Md} = [q_{M1d}, q_{M2d}]^T\) is given as follows.
\[
\begin{align*}
q_{M1d} &= \begin{cases} 
\frac{\pi}{2800} t^2 & (0 \leq t < 20) \\
\frac{\pi}{70} (t - 20) + \frac{\pi}{4} & (20 \leq t < 40) \\
\frac{\pi}{2} - \frac{\pi}{2800} (50 - t)^2 & (40 \leq t \leq 50)
\end{cases}
\end{align*}
\]
\[
\begin{align*}
q_{M2d} &= \begin{cases} 
\frac{\pi}{5600} t^2 & (0 \leq t < 20) \\
\frac{\pi}{140} (t - 20) + \frac{\pi}{44} & (20 \leq t < 40) \\
\frac{\pi}{4} - \frac{\pi}{5600} (50 - t)^2 & (40 \leq t \leq 50)
\end{cases}
\end{align*}
\]

The control parameter is designed as
\[
K_\xi = \begin{bmatrix} 200 & 0 & 0 & 0 \\
0 & 200 & 0 & 0 \\
0 & 0 & 150 & 0 \\
0 & 0 & 0 & 150 \end{bmatrix}
\] (50)

where the parameter \(K_\xi\) satisfies the constraints (40)-(43).

First, \(K_{\xi1}\) and \(K_{\xi2}\) is selected in order to be more than or equal to \(\frac{\pi}{4}\). Next, if \(\gamma = 0.9\) is required, then \(K_{\xi4}\) has to be \(K_{\xi4} \geq 6.5\). In this simulation, \(K_{\xi4}\) is designed as \(K_{\xi4} = 150\). Finally, \(K_{\xi3}\) is decided by using \(\gamma\) and \(K_{\xi4}\).

It is necessary for \(K_{\xi3}\) to be \(K_{\xi3} \geq 16.9\). \(K_{\xi3}\) is designed as \(K_{\xi4} = 150\).

In case of the external force \(\lambda_M = 0\) and \(\lambda_M\) exists, the simulation results are obtained by using MATLAB. When the external force \(\lambda_M\) exists, The force \(\lambda_2\) which is one of \(\lambda_M = [\lambda_{M1} , \lambda_{M2}]\), is applied to the link 2 as shown in Fig. 3.

The reference trajectories and outputs are shown in Figs. 4-9, where the solid line, dashed line, dot-dashed line represent the references, outputs and outputs with external force, respectively. When \(\lambda_M = 0\), the simulation results shows that the mobile manipulator can track along the reference trajectory. In case that the external force exists, the trajectories of \(q_{M1}\) and \(q_{M2}\) are affected by the external force. However, the error vector \(\xi\) is attenuated for the external for the \(\lambda_M\) from Fig. 8-9.
6. CONCLUSIONS

The passivity-based position and force control for the mobile manipulator was considered. By using Lyapunov function, it was shown that the stability of the mobile manipulator can be guaranteed. The design condition to compensate external forces was proposed. Finally, it was indicated that the mobile manipulator can track reference trajectories by utilizing MATLAB in the simulation.

REFERENCES
