第 23 章 : 対称3相交流回路

23.2 対称3相交流回路

23.3 対称3相交流電圧のY-Δ変換

23.5 3相負荷インピーダンスのY-Δ変換

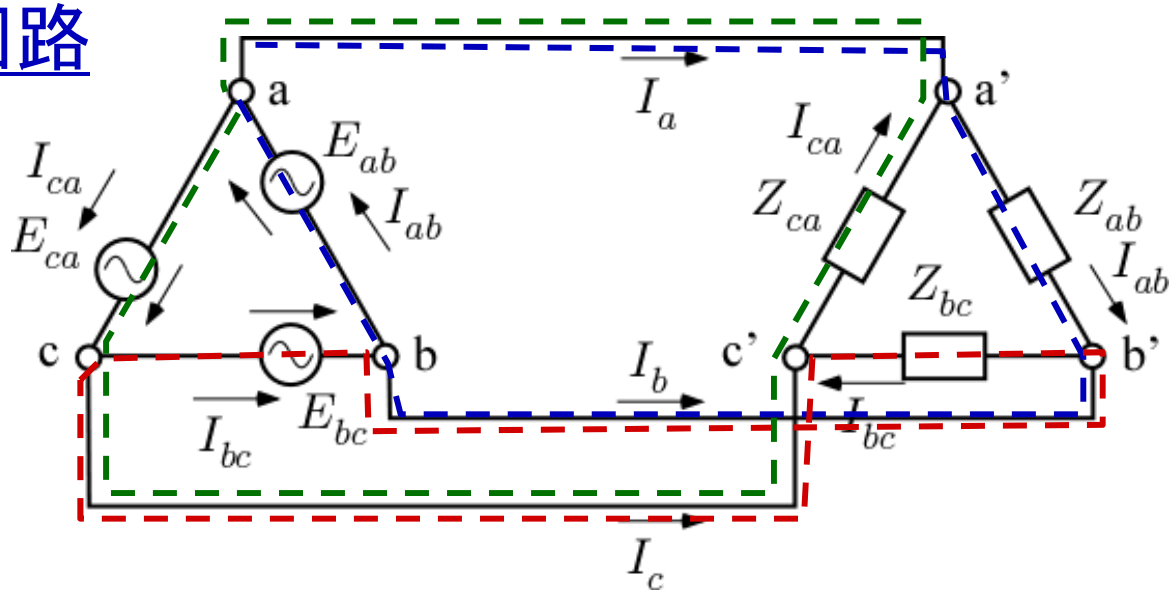
キーワード：Δ-Δ結線，Y-Δ変換，Δ-Y変換

学習目標：平衡Δ-Δ結線，Y-Δ変換，Δ-Y変換を理解できる。

23 対称3相交流回路

23.2 対称3相交流回路

(2) Δ-Δ結線



$$E_{ab} = Z_{ab} I_{ab} \quad (1)$$

$$E_{bc} = Z_{bc} I_{bc} \quad (2)$$

$$E_{ca} = Z_{ca} I_{ca} \quad (3)$$

よって

$$E_{ab} + E_{bc} + E_{ca} = E\angle 0^\circ + E\angle -120^\circ + E\angle -240^\circ$$

$$= E + E\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) + E\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = 0$$

$$E_{ab} + E_{bc} + E_{ca} = 0$$

$$I_{ab} = I\angle 0^\circ$$
$$I_{bc} = I\angle -120^\circ$$
$$I_{ca} = I\angle -240^\circ$$

のとき

$$I_a = I_{ab} - I_{ca}$$
$$= I\angle 0^\circ - I\angle -240^\circ \ = I - I\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = I\left(\frac{3}{2} - j\frac{\sqrt{3}}{2}\right)$$
$$= I\sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}\angle -30^\circ \ = I\sqrt{\frac{12}{4}}\angle -30^\circ$$
$$= \sqrt{3}I\angle -30^\circ$$

$$I_b = I_{bc} - I_{ab}$$

$$= I\angle -120^\circ - I\angle 0^\circ \;= I\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) - I \;= I\left(-\frac{3}{2} - j\frac{\sqrt{3}}{2}\right)$$

$$= I\sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}\angle -150^\circ \;= I\sqrt{\frac{12}{4}}\angle -150^\circ$$

$$= \sqrt{3}I\angle -150^\circ$$

$$I_c = I_{ca} - I_{bc}$$

$$= I\angle -240^\circ - I\angle -120^\circ \;= I\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) - I\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)$$

$$= j\sqrt{3}I$$

$$= I\angle -270^\circ$$

(線電流) = (相電流) $\times\sqrt{3}\angle -30^\circ$

(相電流)

$$I_{ab} = I$$

$$I_{bc} = I\angle -\frac{2}{3}\pi = I\angle -120^\circ$$

$$I_{ca} = I\angle -\frac{4}{3}\pi = I\angle -240^\circ$$

(線電流)

$$I_a = \sqrt{3}I\angle -30^\circ$$

$$I_b = \sqrt{3}I\angle -150^\circ$$

$$I_c = \sqrt{3}I\angle -270^\circ$$

(相電圧) = (線電圧)

【問題26.5】

$$I_{ab} = \frac{50\angle 0^\circ}{5} = 10\angle 0^\circ$$

$$I_{bc} = \frac{50\angle -120^\circ}{5} = 10\angle -120^\circ$$

$$I_{ca} = \frac{50\angle -240^\circ}{5} = 10\angle -240^\circ$$

$$I'_{ab} = I_{ab} = 10\angle 0^\circ$$

$$I'_{bc} = I_{bc} = 10\angle -120^\circ$$

$$I'_{ca} = I_{ca} = 10\angle -240^\circ$$

$$I_a = \sqrt{3} I_{ab} \angle -30^\circ = 10\sqrt{3}\angle -30^\circ$$

$$I_b = \sqrt{3} I_{bc} \angle -30^\circ = 10\sqrt{3}\angle -150^\circ$$

$$I_c = \sqrt{3} I_{ca} \angle -30^\circ = 10\sqrt{3}\angle -270^\circ$$

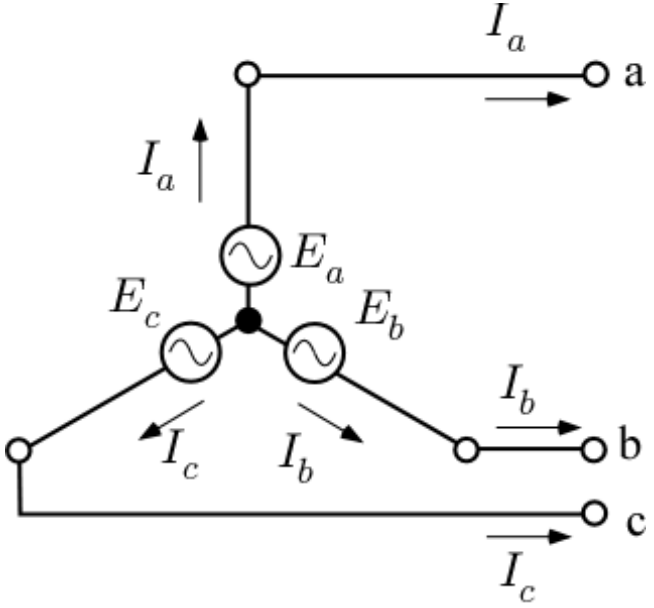
23 対称3相交流回路

23.3 対称3相交流電圧のY-Δ変換

$$E_{ab} = E_a - E_b$$

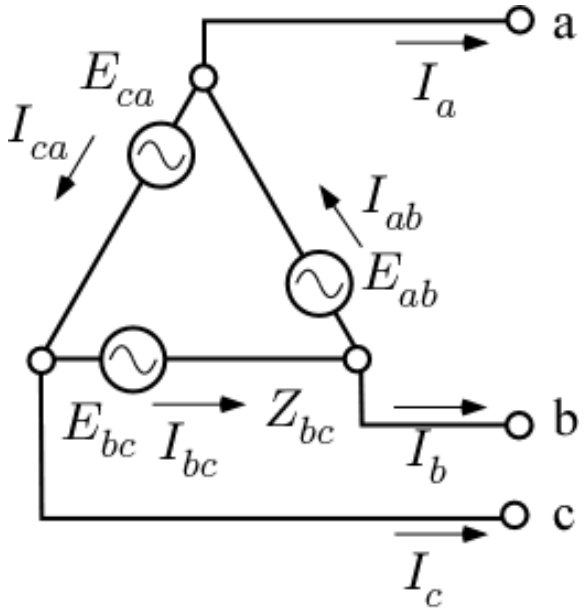
$$E_{bc} = E_b - E_c$$

$$E_{ca} = E_c - E_a$$



Δ結線の電圧 $= \sqrt{3}$ |Y結線の電圧|$\angle 30°$

大きさは $\sqrt{3}$ 倍，位相は $30°$ 進む



（証明）

$$E_a = E\angle 0^\circ$$

$$E_b = E\angle -120^\circ$$

$$E_c = E\angle -240^\circ$$

のとき

$$E_{ab} = E - E\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) = E\left(\frac{3}{2} + j\frac{\sqrt{3}}{2}\right) = \sqrt{3}E\angle 30^\circ$$

$$E_{bc} = E\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) - E\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = E\left(-j\sqrt{3}\right) = \sqrt{3}E\angle -90^\circ$$

$$E_{ca} = E\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) - E = E\left(-\frac{3}{2} + j\frac{\sqrt{3}}{2}\right) = \sqrt{3}E\angle -210^\circ$$

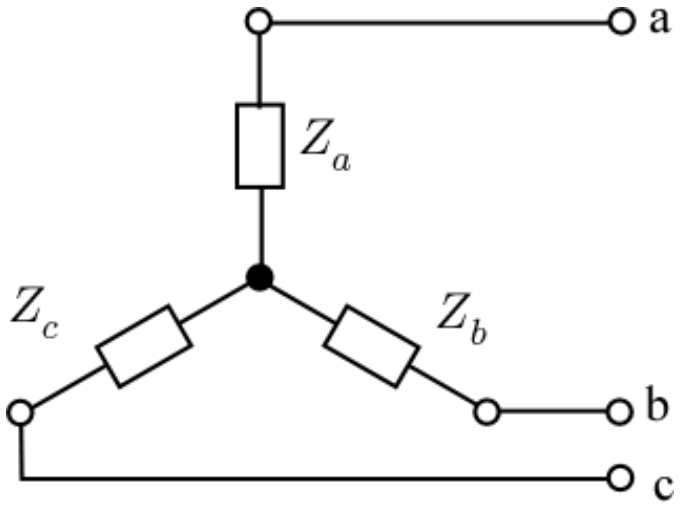
23 対称3相交流回路

23.5 3相負荷インピーダンスのY-Δ変換

$$Z_a = \frac{Z_{ab}Z_{ca}}{Z_{ab}+Z_{bc}+Z_{ca}}$$

$$Z_b = \frac{Z_{bc}Z_{ab}}{Z_{ab}+Z_{bc}+Z_{ca}}$$

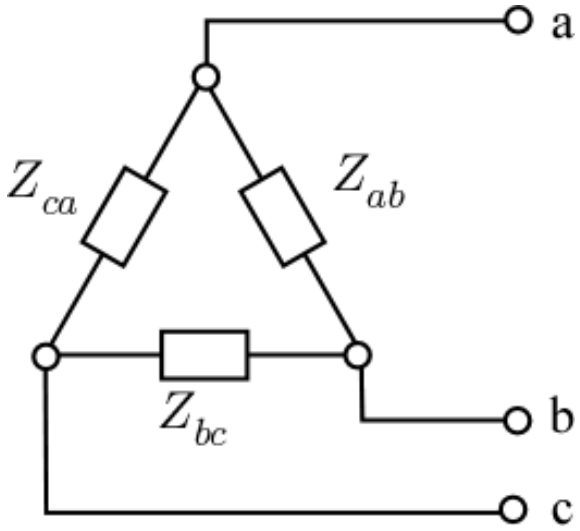
$$Z_c = \frac{Z_{ca}Z_{bc}}{Z_{ab}+Z_{bc}+Z_{ca}}$$



$$Z_{ab} = \frac{Z_aZ_b+Z_bZ_c+Z_cZ_a}{Z_c}$$

$$Z_{bc} = \frac{Z_aZ_b+Z_bZ_c+Z_cZ_a}{Z_a}$$

$$Z_{ca} = \frac{Z_aZ_b+Z_bZ_c+Z_cZ_a}{Z_b}$$

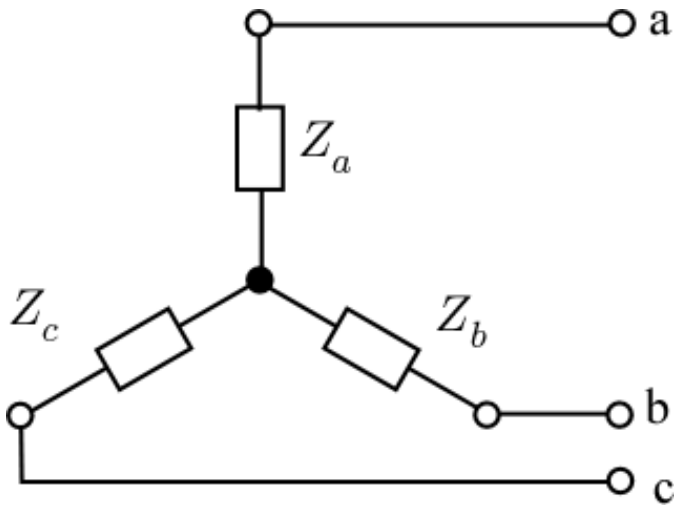


$Z_{ab} = Z_{bc} = Z_{ca} = Z$ のとき

$$Z_a = \frac{Z_{ab}Z_{ca}}{Z_{ab} + Z_{bc} + Z_{ca}} = \frac{Z}{3}$$

$$Z_b = \frac{Z_{bc}Z_{ab}}{Z_{ab} + Z_{bc} + Z_{ca}} = \frac{Z}{3}$$

$$Z_c = \frac{Z_{ca}Z_{bc}}{Z_{ab} + Z_{bc} + Z_{ca}} = \frac{Z}{3}$$

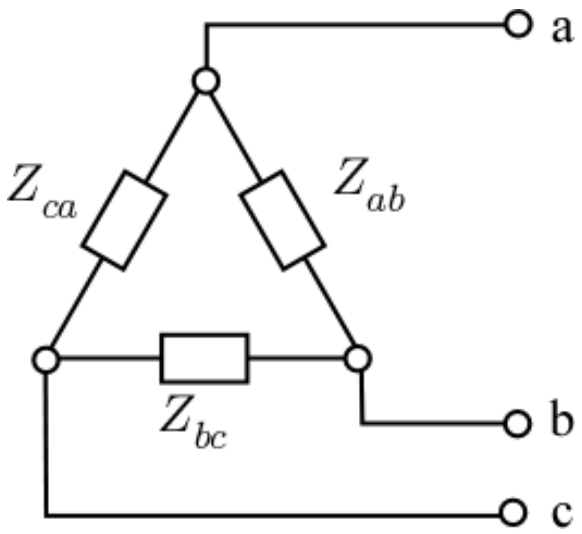


$Z_a = Z_b = Z_c = Z$ のとき

$$Z_{ab} = \frac{Z_aZ_b + Z_bZ_c + Z_cZ_a}{Z_c} = 3Z$$

$$Z_{bc} = \frac{Z_aZ_b + Z_bZ_c + Z_cZ_a}{Z_a} = 3Z$$

$$Z_{ca} = \frac{Z_aZ_b + Z_bZ_c + Z_cZ_a}{Z_b} = 3Z$$



（証明）

a – b 間のインピーダンス

$$Z_{ab} = Z_a + Z_b = \frac{1}{\frac{1}{Z_{ab}} + \frac{1}{Z_{ca}+Z_{bc}}} = \frac{Z_{ab}(Z_{bc}+Z_{ca})}{Z_{ab}+Z_{bc}+Z_{ca}} \qquad (1)$$

b – c 間のインピーダンス

$$Z_{bc} = Z_b + Z_c = \frac{1}{\frac{1}{Z_{bc}} + \frac{1}{Z_{ab}+Z_{ca}}} = \frac{Z_{bc}(Z_{ca}+Z_{ab})}{Z_{ab}+Z_{bc}+Z_{ca}} \qquad (2)$$

c – a 間のインピーダンス

$$Z_{ca} = Z_c + Z_a = \frac{1}{\frac{1}{Z_{ca}} + \frac{1}{Z_{ab}+Z_{bc}}} = \frac{Z_{ca}(Z_{ab}+Z_{bc})}{Z_{ab}+Z_{bc}+Z_{ca}} \qquad (3)$$

$(1) \sim (3)$ 式を足し合わせる

$$2(Z_a + Z_b + Z_c) = 2\frac{Z_{ab}Z_{bc} + Z_{bc}Z_{ca} + Z_{ca}Z_{ab}}{Z_{ab}+Z_{bc}+Z_{ca}}$$

$$Z_a + Z_b + Z_c = \frac{Z_{ab}Z_{bc} + Z_{bc}Z_{ca} + Z_{ca}Z_{ab}}{Z_{ab}+Z_{bc}+Z_{ca}} \qquad (4)$$

(4) 式 - (2) 式

$$Z_a = \frac{Z_{ab}Z_{ca}}{Z_{ab} + Z_{bc} + Z_{ca}}$$

(4) 式 - (3) 式

$$Z_b = \frac{Z_{bc}Z_{ab}}{Z_{ab} + Z_{bc} + Z_{ca}}$$

(4) 式 - (1) 式

$$Z_c = \frac{Z_{ca}Z_{bc}}{Z_{ab} + Z_{bc} + Z_{ca}}$$

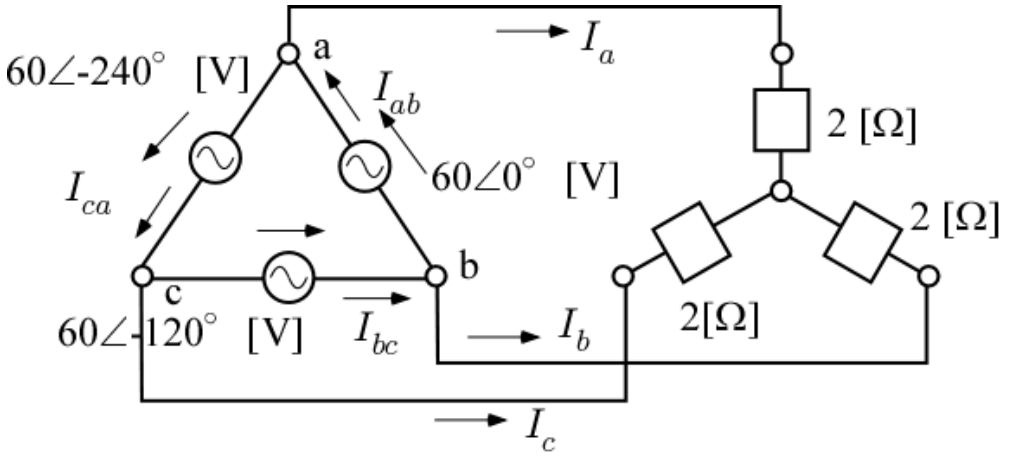
【問題 26.6】

電源をY型に変換

$$V_a' = \frac{60}{\sqrt{3}} \angle -30^\circ = 20\sqrt{3} \angle -30^\circ$$

$$V_b' = \frac{60}{\sqrt{3}} \angle -150^\circ = 20\sqrt{3} \angle -150^\circ$$

$$V_c' = \frac{60}{\sqrt{3}} \angle -270^\circ = 20\sqrt{3} \angle -270^\circ$$

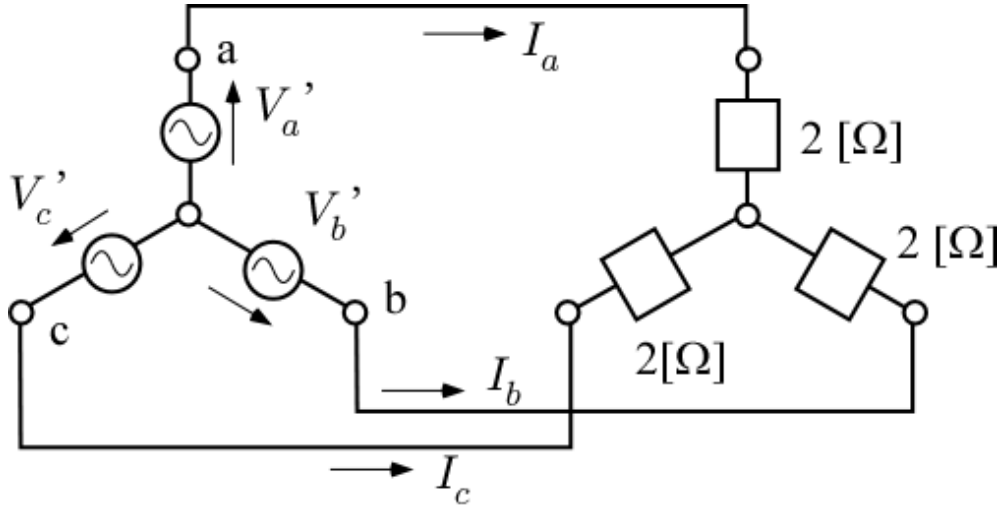


電流

$$I_a = \frac{V_a'}{2} = 10\sqrt{3} \angle -30^\circ$$

$$I_b = \frac{V_b'}{2} = 10\sqrt{3} \angle -150^\circ$$

$$I_c = \frac{V_c'}{2} = 10\sqrt{3} \angle -270^\circ$$



$$I_a = I_{ab} - I_{ca}$$

$I_{ab} = I\angle 0^\circ,\ I_{ca} = I\angle -240^\circ$ と仮定

$$I_a = I - I\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \frac{I}{2}\left(3 - j\sqrt{3}\right) = \sqrt{3}I\angle -30^\circ$$

I_a は I_{ab} より大きさが $\sqrt{3}$ 倍，位相が 30° 遅れる

I_{ab} は I_a より大きさが $\frac{1}{\sqrt{3}}$ 倍，位相が 30° 進む

$$I_{ab} = I_a\frac{1}{\sqrt{3}}\angle 30^\circ = 10\angle 0^\circ$$

$$I_{bc} = I_b\frac{1}{\sqrt{3}}\angle 30^\circ = 10\angle -120^\circ$$

$$I_{ca} = I_c\frac{1}{\sqrt{3}}\angle 30^\circ = 10\angle -240^\circ$$

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キーワード：Δ-Δ結線，Y-Δ変換，Δ-Y変換

学習目標：平衡Δ-Δ結線，Y-Δ変換，Δ-Y変換を理解できる。