

第5章：ラプラス変換とその応用

5.4 微分方程式の解

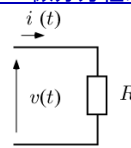
キーワード：s回路法

学習目標：s回路法を用いて複雑な回路の過渡応答を解けるようになる。

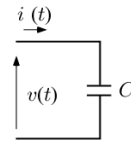
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5 ラプラス変換とその応用

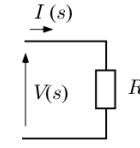
5.4 微分方程式の解



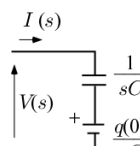
$$v(t) = Ri(t)$$



$$v(t) = \frac{1}{C} \int i(t) dt$$



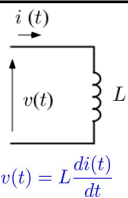
$$V(s) = RI(s)$$



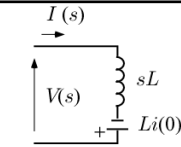
$$\frac{q(0)}{sC} = \frac{Cv_C(0)}{sC} = \frac{v_C(0)}{s}$$

$$V(s) = \frac{1}{sC}I(s) + \frac{1}{sC}q(0)$$

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$$v(t) = L \frac{di(t)}{dt}$$



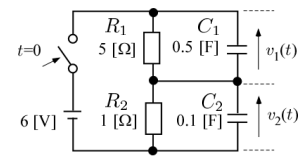
$$V(s) = L(sI(s) - i(0))$$

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初期値 = 0 のときに有効

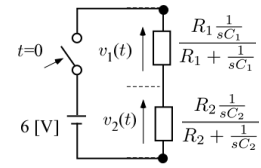
$$\frac{R_1 \frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}} = \frac{R_1}{sR_1C_1 + 1}$$

$$\frac{R_2 \frac{1}{sC_2}}{R_2 + \frac{1}{sC_2}} = \frac{R_2}{sR_2C_2 + 1}$$



分圧より

$$\begin{aligned} V_1(s) &= \frac{\frac{R_1 \frac{1}{sC_1}}{sR_1C_1 + 1}}{\frac{R_1 \frac{1}{sC_1}}{sR_1C_1 + 1} + \frac{R_2}{sR_2C_2 + 1}} \times \frac{6}{s} \\ &= \frac{\frac{5}{2.5s+1}}{\frac{5}{2.5s+1} + \frac{1}{0.1s+1}} \times \frac{6}{s} \\ &= \frac{5(0.1s+1)}{5(0.1s+1) + (2.5s+1)} \times \frac{6}{s} \\ &= \frac{0.5s+5}{3s+6} \times \frac{6}{s} \end{aligned}$$



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$$\begin{aligned} V_1(s) &= \frac{0.5s+5}{3s+6} \times \frac{6}{s} \\ &= \frac{2(0.5s+5)}{s(s+2)} \\ &= \frac{s+10}{s(s+2)} \\ &= \frac{5}{s} - \frac{4}{s+2} \end{aligned}$$

逆ラプラス変換

$$v_1(t) = 5 - 4e^{-2t}$$

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RL回路

$$\begin{aligned} RI(s) + L(sI(s) - i(0)) &= \frac{V}{s} \\ (sL + R)I(s) &= \frac{V}{s} + Li(0) \end{aligned}$$

よって

$$\begin{aligned} I(s) &= \frac{V}{s(sL + R)} + \frac{Li(0)}{sL + R} \\ &= \frac{V}{R} \frac{1}{s} - \frac{V}{R} \frac{1}{s + \frac{R}{L}} + \frac{i(0)}{s + \frac{R}{L}} \end{aligned}$$

逆ラプラス変換

$$\begin{aligned} i(t) &= \frac{V}{R} - \frac{V}{R} e^{-\frac{R}{L}t} + i(0)e^{-\frac{R}{L}t} \\ &= \frac{V}{R} \left(1 - e^{-\frac{R}{L}t}\right) + i(0)e^{-\frac{R}{L}t} \end{aligned}$$

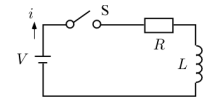
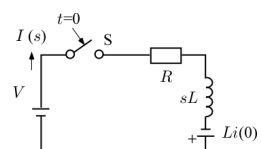


図2 RL回路



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RC回路

$$RI(s) + \frac{1}{sC}I(s) + \frac{q(0)}{sC} = \frac{V}{s}$$

$$\left(R + \frac{1}{Cs}\right)I(s) = \frac{V}{s} - \frac{q(0)}{Cs}$$

$$\frac{RCs+1}{Cs}I(s) = \frac{V}{s} - \frac{q(0)}{Cs}$$

よって

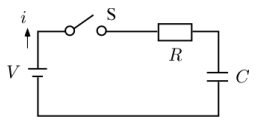
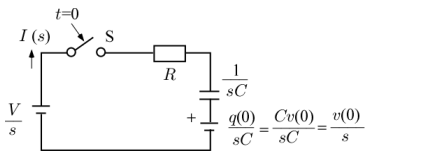
$$I(s) = \frac{CV}{RCs+1} - \frac{q(0)}{RCs+1} = \frac{\frac{1}{R}V}{s + \frac{1}{RC}} - \frac{\frac{1}{RC}q(0)}{s + \frac{1}{RC}}$$


図1 RC回路



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逆ラプラス変換により

$$I(s) = \frac{\frac{1}{R}V}{s + \frac{1}{RC}} - \frac{\frac{1}{RC}q(0)}{s + \frac{1}{RC}}$$

$$i(t) = \frac{V}{R}e^{-\frac{t}{RC}} - \frac{q(0)}{RC}e^{-\frac{t}{RC}}$$

$$= \frac{1}{R}\left(V - \frac{q_0}{C}\right)e^{-\frac{t}{RC}}$$

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ドリル【問題44.2】(2)

ドリル【問題44.3】(2)

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5.4 微分方程式の解

キーワード：RC回路, RL回路

学習目標：ラプラス変換を用いてRC回路, RL回路の過渡応答を解けるようになる。

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