

第1章：1端子対回路

1.5 リアクタンス関数の合成

キーワード： Foster展開(部分分数展開)

学習目標： Foster展開でリアクタンス関数を合成することができる。

1 1端子対回路

1.5 リアクタンス関数の合成

Foster展開(部分分数展開) インピーダンス

$$\begin{aligned} Z(s) &= H \frac{(s^2 + \omega_1^2)(s^2 + \omega_3^2) \cdots (s^2 + \omega_{2n+1}^2)}{s(s^2 + \omega_2^2) \cdots (s^2 + \omega_{2n}^2)} \\ &= \frac{h_0}{s} + \sum_{k=1}^n \frac{h_{2k}s}{s^2 + \omega_{2k}^2} + h_\infty s \end{aligned}$$

$$h_0 = [sZ(s)]_{s=0} \geq 0$$

$$h_\infty = \left[\frac{1}{s} Z(s) \right]_{s=\infty} \geq 0$$

$$h_{2k} = \left[\frac{s^2 + \omega_{2k}^2}{s} Z(s) \right]_{s^2 = -\omega_{2k}^2} \geq 0 \quad (k = 1, 2, \cdots, n)$$

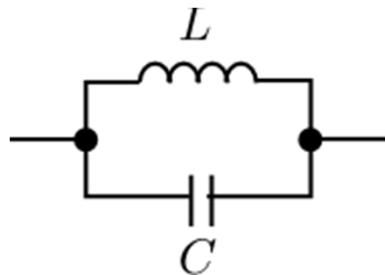
$$Z(s) = \frac{h_0}{s} + \sum_{k=1}^n \frac{h_{2k}s}{s^2 + \omega_{2k}^2} + \frac{h_\infty s}{s}$$

キャパシタンス
インダクタンス

$$\frac{h_0}{s} = \frac{1}{sC_0} \quad C_0 = \frac{1}{h_0} = \frac{1}{[sZ(s)]_{s=0}}$$

$$h_\infty s = sL_\infty \quad L_\infty = h_\infty = \left[\frac{1}{s} Z(s) \right]_{s=\infty}$$

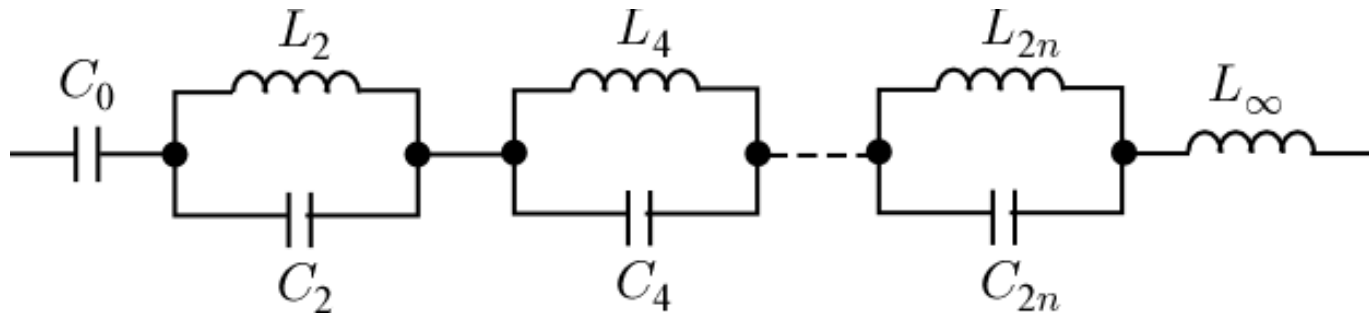
$$Z_{2k}(s) = \frac{h_{2k}s}{s^2 + \omega_{2k}^2} = \frac{sL_{2k} \cdot \frac{1}{sC_{2k}}}{sL_{2k} + \frac{1}{sC_{2k}}} = \frac{\frac{s}{C_{2k}}}{s^2 + \frac{1}{L_{2k}C_{2k}}}$$



$$Z = \frac{sL \frac{1}{sC}}{sL + \frac{1}{sC}} = \frac{sL}{s^2 LC + 1} = \frac{s \frac{1}{C}}{s^2 + \frac{1}{LC}}$$

$$C_{2k} = \frac{1}{h_{2k}} = \frac{1}{\left[\frac{s^2 + \omega_{2k}^2}{s} Z(s) \right]_{s^2 = -\omega_{2k}^2}}$$

$$L_{2k} = \frac{h_{2k}}{\omega_{2k}^2} = \frac{1}{\omega_{2k}^2} \left[\frac{s^2 + \omega_{2k}^2}{s} Z(s) \right]_{s^2 = -\omega_{2k}^2}$$



【例】第1章【16】(1)

リアクタンス関数をFoster形で2種類ずつ実現せよ。

$$Z_1(s) = \frac{(s^2 + 1)(s^2 + 9)}{s(s^2 + 4)}$$

$$Z_1(s) = \frac{h_0}{s} + \frac{h_2 s}{s^2 + 4} + h_\infty s \quad \text{とおく}$$

$$h_0 = [sZ_1(s)]_{s=0} = \left[\cancel{s} \frac{(s^2 + 1)(s^2 + 9)}{\cancel{s}(s^2 + 4)} \right]_{s=0} = \frac{(0 + 1)(0 + 9)}{(0 + 4)} = \frac{9}{4}$$

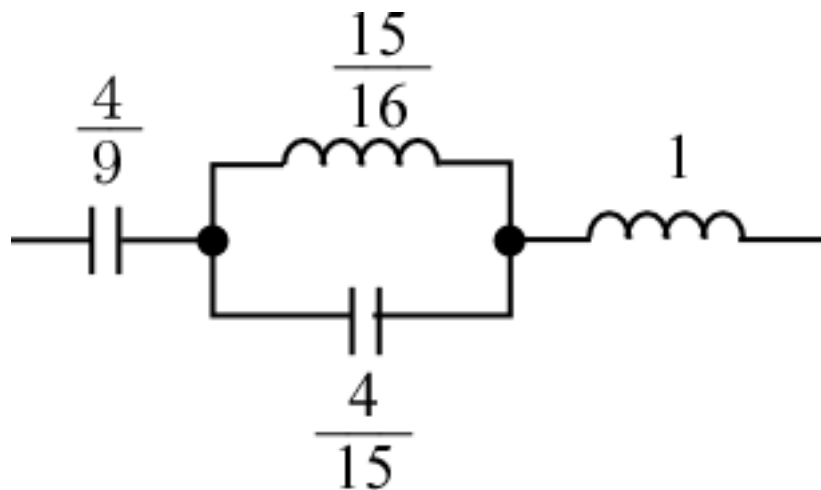
$$\begin{aligned} h_2 &= \left[\frac{s^2 + 4}{s} Z(s) \right]_{s^2=-4} = \left[\frac{\cancel{s^2 + 4}}{s} \frac{(s^2 + 1)(s^2 + 9)}{s(\cancel{s^2 + 4})} \right]_{s^2=-4} \\ &= \frac{(-4 + 1)(-4 + 9)}{-4} = \frac{-3 \times 5}{-4} = \frac{15}{4} \end{aligned}$$

$$h_{\infty} = \left[\frac{1}{s} Z(s) \right]_{s=\infty} = \left[\frac{1}{s} \frac{(s^2 + 1)(s^2 + 9)}{s(s^2 + 4)} \right]_{s=\infty}$$

$$= \left[\frac{s^4 + 10s^2 + 9}{s^4 + 4s^2} \right]_{s=\infty} = \left[\frac{s^4}{s^4} \right]_{s=\infty} = 1$$

$$Z_1(s) = \frac{9}{4s} + \frac{15s}{4(s^2 + 4)} + s = \frac{1}{\frac{4}{9}s} + \frac{15s}{4(s^2 + 4)} + s$$

$$\frac{15s}{4(s^2 + 4)} = \frac{1}{\frac{4}{15s}(s^2 + 4)} = \frac{1}{\frac{4}{15}s + \frac{16}{15s}} = \frac{1}{\frac{4}{15}s + \frac{1}{\frac{15}{16}s}}$$



Foster展開(部分分数展開) アドミタンス

$$Y(s) = H \frac{s(s^2 + \omega_2^2) \cdots (s^2 + \omega_{2n}^2)}{(s^2 + \omega_1^2)(s^2 + \omega_3^2) \cdots (s^2 + \omega_{2n+1}^2)}$$
$$= \frac{h_0}{s} + \sum_{k=0}^n \frac{h_{2k+1}s}{s^2 + \omega_{2k+1}^2} + h_\infty s$$

$$h_0 = [sY(s)]_{s=0} \geq 0$$

$$h_\infty = \left[\frac{1}{s} Y(s) \right]_{s=\infty} \geq 0$$

$$h_{2k} = \left[\frac{s^2 + \omega_{2k+1}^2}{s} Y(s) \right]_{s^2 = -\omega_{2k+1}^2} \geq 0 \quad (k = 0, 1, 2, \dots, n)$$

$$Y(s) = \underbrace{\frac{h_0}{s}}_{\substack{\text{インダク} \\ \text{タンス}}} + \sum_{k=0}^n \frac{h_{2k+1}s}{\underbrace{s^2 + \omega_{2k+1}^2}_{\substack{\text{キャパ} \\ \text{シタンス}}} + h_\infty s$$

$$\frac{h_0}{s} = \frac{1}{sL_0} \quad L_0 = \frac{1}{h_0} = \frac{1}{[sY(s)]_{s=0}}$$

$$h_\infty s = sC_\infty \quad C_\infty = h_\infty = \left[\frac{1}{s} Y(s) \right]_{s=\infty}$$

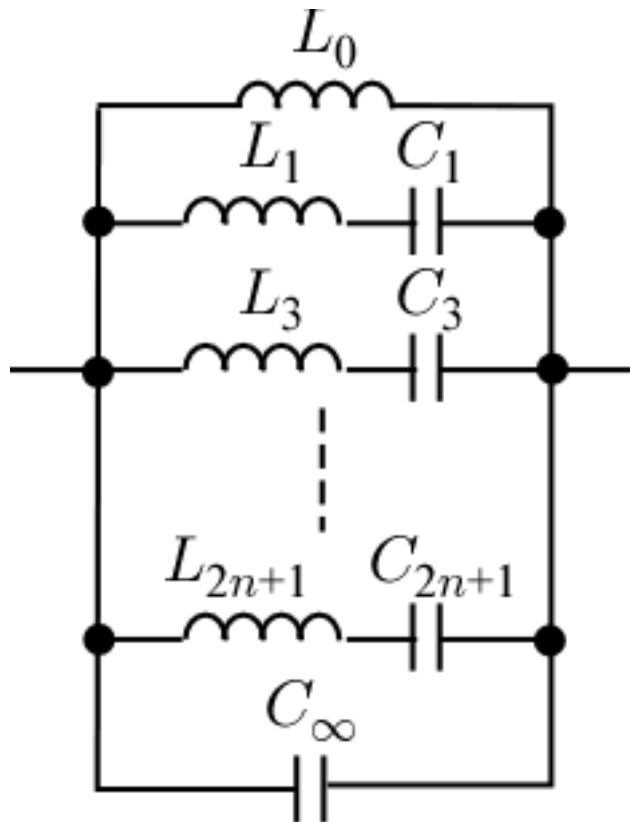
$$Y_{2k+1}(s) = \frac{\underline{h_{2k+1}s}}{s^2 + \underline{\omega_{2k+1}^2}} = \frac{1}{sL_{2k+1} + \frac{1}{sC_{2k+1}}} = \frac{\frac{s}{\underline{L_{2k+1}}}}{s^2 + \frac{1}{\underline{L_{2k+1}C_{2k+1}}}}$$



$$Y = \frac{1}{sL + \frac{1}{sC}}$$

$$L_{2k+1} = \frac{1}{h_{2k+1}} = \left[\frac{s}{(s^2 + \omega_{2k+1}^2)Y(s)} \right]_{s^2 = -\omega_{2k+1}^2}$$

$$C_{2k+1} = \frac{h_{2k+1}}{\omega_{2k+1}^2} = \frac{1}{\omega_{2k+1}^2} \left[\frac{s^2 + \omega_{2k+1}^2}{s} Y(s) \right]_{s^2 = -\omega_{2k+1}^2}$$



【例】第1章【16】(1)

$$Y_1(s) = \frac{1}{Z_1(s)} = \frac{s(s^2 + 4)}{(s^2 + 1)(s^2 + 9)}$$

$$Y_1(s) = \frac{h_2 s}{s^2 + 1} + \frac{h_4 s}{s^2 + 9} + h_\infty s + \frac{h_0}{s} \quad \text{とおく}$$

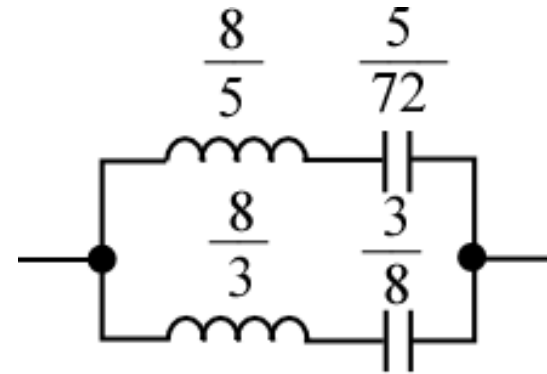
$$h_0 = \left[s \frac{s(s^2 + 4)}{(s^2 + 1)(s^2 + 9)} \right]_{s=0} = 0$$

$$h_\infty = \left[\frac{1}{s} \frac{s(s^2 + 4)}{(s^2 + 1)(s^2 + 9)} \right]_{s=\infty} = \left[\frac{s^2}{s^4} \right]_{s=\infty} = 0$$

$$h_2 = \left[\frac{\cancel{s^2 + 1}}{s} \frac{s \cancel{(s^2 + 4)}}{(\cancel{s^2 + 1})(s^2 + 9)} \right]_{s^2 = -1} = \frac{-1 + 4}{-1 + 9} = \frac{3}{8}$$

$$h_4 = \left[\frac{\cancel{s^2 + 9}}{s} \frac{s \cancel{(s^2 + 4)}}{(s^2 + 1)(\cancel{s^2 + 9})} \right]_{s^2 = -9} = \frac{-9 + 4}{-9 + 1} = \frac{-5}{-8} = \frac{5}{8}$$

$$\begin{aligned}
Y_1(s) &= \frac{3s}{8(s^2 + 1)} + \frac{5s}{8(s^2 + 9)} \\
&= \frac{1}{\frac{8}{3s}(s^2 + 1)} + \frac{1}{\frac{8}{5s}(s^2 + 9)} \\
&= \frac{1}{\frac{8}{3}s + \frac{8}{3s}} + \frac{1}{\frac{8}{5}s + \frac{72}{5s}} \\
&= \frac{1}{\frac{8}{3}s + \frac{1}{\frac{3}{8}s}} + \frac{1}{\frac{8}{5}s + \frac{1}{\frac{5}{72}s}}
\end{aligned}$$



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