第 1 章：1端子対回路

1.6 RL 1端子対回路

1.7 RC 1端子対回路

キーワード： Foster展開，Cauer展開

学習目標：RL回路，RC回路をFoster展開やCauer展開で合成することができる。

1 1端子対回路

1.6 RL 1端子対回路

RL直列回路

$$Z_{RL}(s) = R + sL$$

LC直列回路

$$Z_{LC}(s) = \frac{h_0}{s} + \sum_{k=1}^{n} \frac{h_{2k}s}{s^2 + \omega_{2k}^2} + h_\infty s$$

$$Z_{RL}(s) = h_0 + \sum_{k=1}^{n} h_k - \sum_{k=1}^{n} \frac{\omega_k h_k}{s + \omega_k} + h_\infty s$$

- 極が負の実軸上および無限遠点にのみ存在

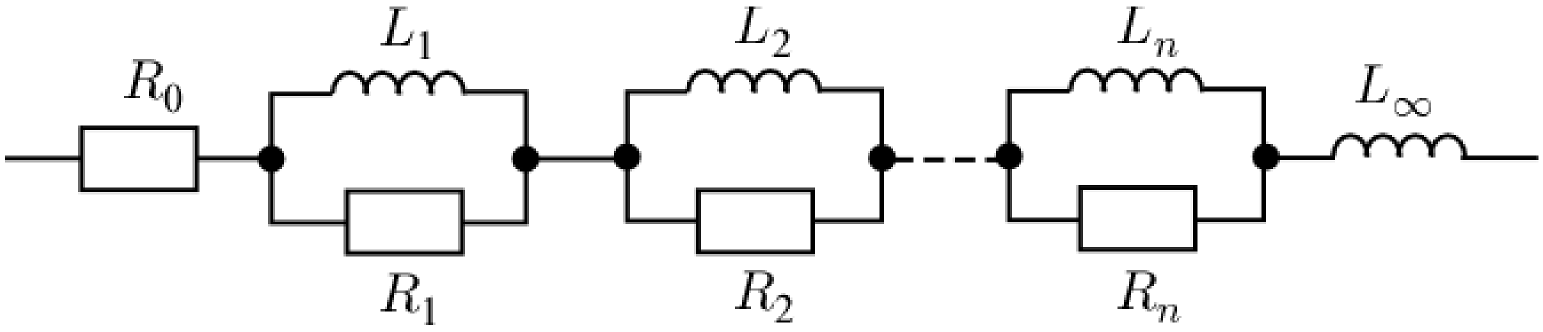
  $Z_{RL}(s) = \infty$ となるとき $s = -\omega_k,\ \infty$

- 無限遠点を除く極の留数は負の値をとる

$$[(s + \omega_k) Z_{RL}(s)]_{s=-\omega_k} = -\sum_{k=1}^{n} \omega_k h_k$$

Foster形

$$Z_{RL}(s) = h_0 + \sum_{k=1}^{n} \frac{h_k s}{s + \omega_k} + h_\infty s$$



$$R_0 = h_0 = [Z_{RL}(s)]_{s=0}$$

$$L_\infty = \left[\frac{Z_{RL}(s)}{s}\right]_{s=\infty}$$

$$R_k = h_k = \left[\frac{s+\omega_k}{s} Z_{RL}(s)\right]_{s=-\omega_k}$$

$$L_k = \frac{R_k}{\omega_k} = \frac{1}{\omega_k}\left[\frac{s+\omega_k}{s} Z_{RL}(s)\right]_{s=-\omega_k}$$

Cauer形

$$Z_{RL}(s) = h_0 + \sum_{k=1}^{n} \frac{h_k s}{s + \omega_k} + h_\infty s$$

$$= a_0 s + \cfrac{1}{a_1 + \cfrac{1}{a_2 s + \cfrac{1}{a_3 + \cdots}}}$$

$$= b_0 + \cfrac{1}{\frac{b_1}{s} + \cfrac{1}{b_2 + \cfrac{1}{\frac{b_3}{s} + \cdots}}}$$

$$= H \frac{s(s + \omega_1)(s + \omega_3) \cdots (s + \omega_{2n+1})}{(s + \omega_2) \cdots (s + \omega_{2n})}$$

RC直列回路

$$Z_{RC}(s) = R + \frac{1}{sC}$$

$$Z_{RC}(s) = \left[\frac{1}{s}Z_{LC}(s)\right]_{s^2=s} = \frac{h_0}{s} + \sum_{k=1}^{n}\frac{h_k}{s+\omega_k} + h_\infty$$

- $s=0$ の点を含むすべての極が負の実軸上に存在し，**その留数は正**

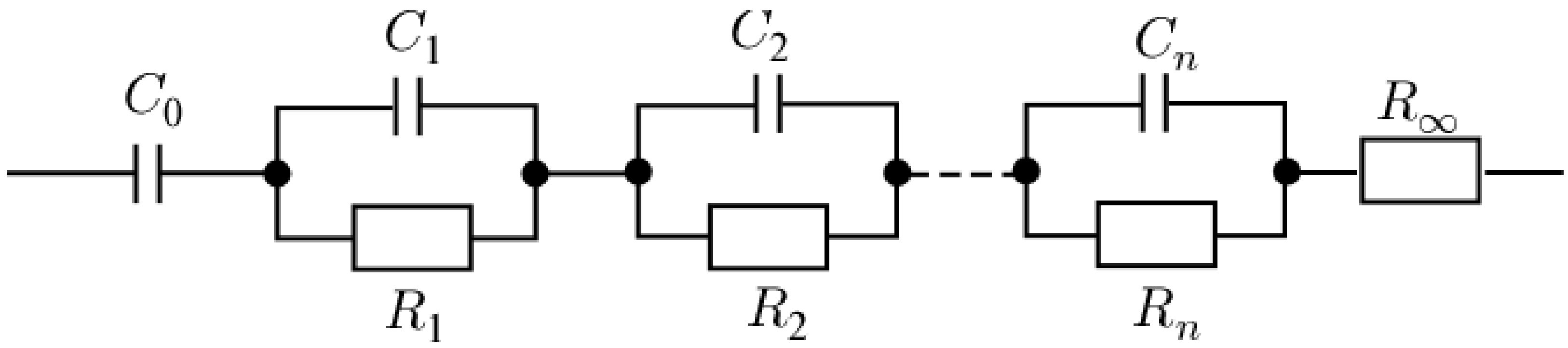
$$\left[(s+\omega_k)Z_{RC}(s)\right]_{s=-\omega_k} = \sum_{k=1}^{n} h_k$$

$$\left[sZ_{RC}(s)\right]_{s=0} = h_0$$

- $s=\infty$ の点は零点または有限な実数値であり，負の実軸上の最右端には極が存在する

Foster形

$$Z_{RC}(s) = \frac{h_0}{s} + \sum_{k=1}^{n} \frac{h_k}{s+\omega_k} + h_\infty$$



$$C_0 = \frac{1}{h_0} = \frac{1}{[sZ_{RC}(s)]_{s=0}}$$

$$R_\infty = h_\infty = [Z_{RC}(s)]_{s=\infty}$$

$$C_k = \frac{1}{h_k} = \frac{1}{[(s+\omega_k)Z_{RL}(s)]_{s=-\omega_k}}$$

$$R_k = \frac{h_k}{\omega_k} = \frac{[(s+\omega_k)Z_{RL}(s)]_{s=-\omega_k}}{\omega_k}$$

Cauer形

$$Z_{RC}(s) = \frac{h_0}{s} + \sum_{k=1}^{n} \frac{h_k}{s+\omega_k} + h_\infty$$

$$= a_0 + \cfrac{1}{a_1 s + \cfrac{1}{a_2 + \cfrac{1}{a_3 s + \cdots}}}$$

$$= \frac{b_0}{s} + \cfrac{1}{b_1 + \cfrac{1}{\frac{b_2}{s} + \cfrac{1}{b_3 + \cdots}}}$$

$$= H \frac{(s+\omega_1)(s+\omega_3)\cdots(s+\omega_{2n+1})}{s(s+\omega_2)\cdots(s+\omega_{2n})}$$

【例】第1章【23】(1)

インピーダンス関数をFoster形で実現せよ。

$$Z_1(s) = \frac{(s+3)(s+5)}{(s+2)(s+4)}, \qquad Y_1(s) = \frac{1}{Z_1(s)}$$

(a) インピーダンスの場合

極の留数は正の値よりRL回路でなく，RC回路

$$\left[(s+2)Z_1(s)\right]_{s=-2} = \frac{(-2+3)(-2+5)}{-2+4} > 0$$

$$\left[(s+4)Z_1(s)\right]_{s=-4} = \frac{(-4+3)(-4+5)}{-4+2} > 0$$

$$Z_1(s) = \frac{h_0}{s} + \frac{h_1}{s+2} + \frac{h_2}{s+4} + h_\infty$$

$$Z_1(s) = \frac{(s+3)(s+5)}{(s+2)(s+4)},$$

$$h_0 = [sZ_1(s)]_{s=0} = 0$$

$$h_\infty = [Z_1(s)]_{s=\infty} = \left[\frac{s^2}{s^2}\right]_{s=\infty} = 1$$

$$h_1 = [(s+2)Z_1(s)]_{s=-2} = \left[\frac{(s+3)(s+5)}{s+4}\right]_{s=-2} = \frac{1\times 3}{2} = \frac{3}{2}$$

$$h_2 = [(s+4)Z_1(s)]_{s=-4} = \left[\frac{(s+3)(s+5)}{s+2}\right]_{s=-4} = \frac{-1\times 1}{-2} = \frac{1}{2}$$

$$Z_1(s) = \frac{\frac{3}{2}}{s+2} + \frac{\frac{1}{2}}{s+4} + 1$$

$$Z_1(s) = \frac{\boxed{\frac{3}{2}}}{s+\boxed{2}} + \frac{\boxed{\frac{1}{2}}}{s+\boxed{4}} + 1$$

$$\frac{3}{2} = \frac{1}{C_1}, \quad 2 = \frac{1}{R_1 C_1}, \quad \frac{1}{2} = \frac{1}{C_2}, \quad 4 = \frac{1}{R_2 C_2}$$

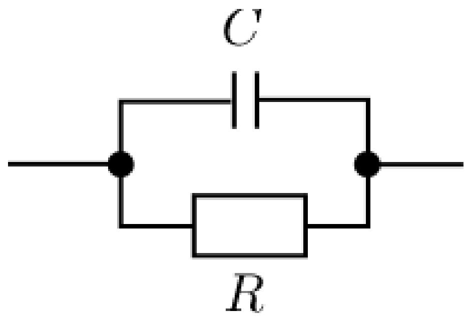
$$= \frac{1}{\boxed{\frac{2}{3}}s + \boxed{\frac{4}{3}}} + \frac{1}{2s+8} + 1$$

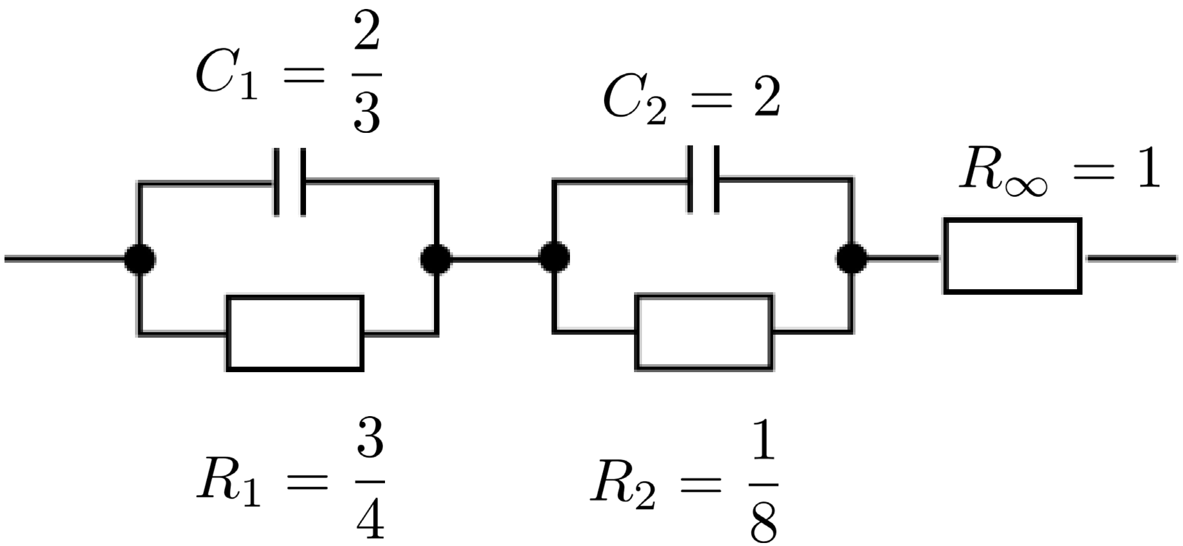
$$\frac{2}{3} \to \frac{1}{C_1}, \quad \frac{4}{3} \to \frac{1}{R_1}$$

$$Z(s) = \frac{\frac{R}{sC}}{R + \frac{1}{sC}} = \frac{R}{sRC+1} = \frac{\frac{1}{C}}{s + \frac{1}{RC}}$$

$$Z = \frac{1}{sC + \frac{1}{R}}$$

上記のどちらで考えてもよい





(b) アドミタンスの場合

$$Y_1(s) = \frac{1}{Z_1(s)} = \frac{(s+2)(s+4)}{(s+3)(s+5)}$$

$$Y_1(s) = h_0 + \frac{h_1 s}{s+3} + \frac{h_2 s}{s+5} + h_\infty s$$

$$h_0 = [Y_1(s)]_{s=0} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}$$

$$h_\infty = \left[\frac{1}{s} Y_1(s)\right]_{s=\infty} = 0$$

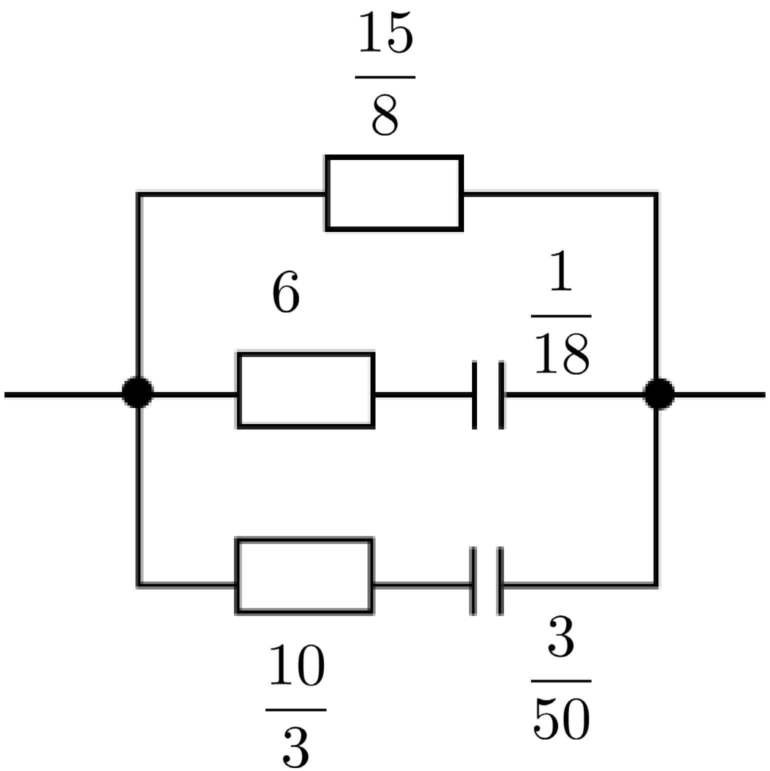
$$h_1 = \left[\frac{s+3}{s} Y_1(s)\right]_{s=-3} = \frac{-1 \times 1}{-3 \times 2} = \frac{1}{6}$$

$$h_2 = \left[\frac{s+5}{s} Y_1(s)\right]_{s=-5} = \frac{-3 \times (-1)}{-5 \times (-2)} = \frac{3}{10}$$

$$Y_1(s) = \frac{8}{15} + \frac{\frac{1}{6} s}{s+3} + \frac{\frac{3}{10} s}{s+5}$$

$$Y_1(s) = \frac{8}{15} + \frac{\frac{1}{6}s}{s+3} + \frac{\frac{3}{10}s}{s+5}$$

$$= \frac{8}{15} + \frac{1}{6 + \frac{18}{s}} + \frac{1}{\frac{10}{3} + \frac{50}{3s}}$$



【例】第1章【23】(1)特別解答1

インピーダンス関数をCauer形で実現せよ。

$$Z_1(s) = \frac{(s+3)(s+5)}{(s+2)(s+4)}$$

$$= \frac{s^2+8s+15}{s^2+6s+8}$$

$$= \boxed{1} + \frac{\boxed{2s+7}}{s^2+6s+8} = 1 + \frac{1}{\frac{s^2+6s+8}{2s+7}}$$

$$= 1 + \frac{1}{\boxed{\frac{1}{2}s} + \frac{\boxed{\frac{5}{2}s+8}}{2s+7}} = 1 + \frac{1}{\frac{1}{2}s + \frac{1}{\frac{2s+7}{\frac{5}{2}s+8}}}$$

$$= 1 + \frac{1}{\frac{1}{2}s + \frac{1}{\frac{2s+7}{\frac{5}{2}s+8}}}$$

$$= 1 + \frac{1}{\frac{1}{2}s + \frac{1}{\boxed{\frac{4}{5}} + \frac{\boxed{\frac{3}{5}}}{\frac{5}{2}s+8}}}$$

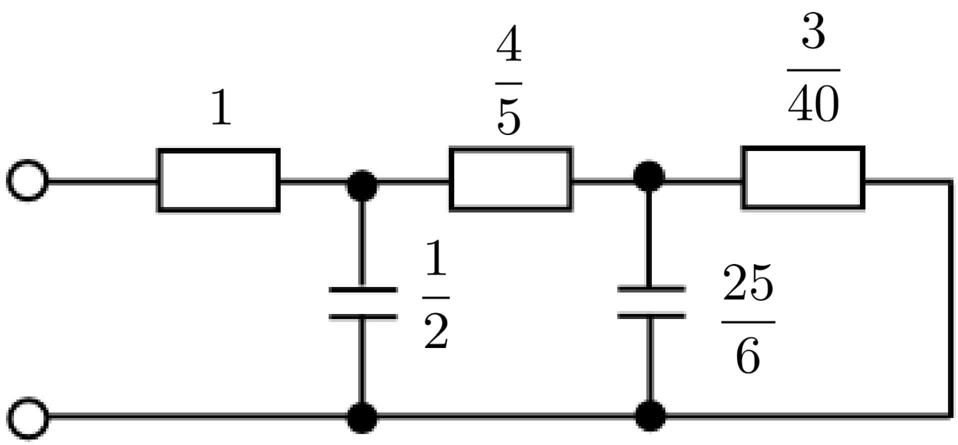
$$\begin{array}{r|l} & \boxed{1} \\ s^2+6s+8 & s^2+8s+15 \\ & s^2+6s+8 \\ \hline & \boxed{2s+7} \end{array}$$

$$\begin{array}{r|l} & \boxed{\frac{1}{2}s} \\ 2s+7 & s^2+6s+8 \\ & s^2+\frac{7}{2}s \\ \hline & \boxed{\frac{5}{2}s+8} \end{array}$$

$$\begin{array}{r|l} & \boxed{\frac{4}{5}} \\ \frac{5}{2}s+8 & 2s+7 \\ & 2s+\frac{32}{5} \\ \hline & \boxed{\frac{3}{5}} \end{array}$$

$$= 1 + \cfrac{1}{\frac{1}{2}s + \cfrac{1}{\frac{4}{5} + \cfrac{\frac{3}{5}}{\frac{5}{2}s + 8}}} = 1 + \cfrac{1}{\frac{1}{2}s + \cfrac{1}{\frac{4}{5} + \cfrac{1}{\frac{25}{6}s + \frac{40}{3}}}}$$

$$= 1 + \cfrac{1}{\frac{1}{2}s + \cfrac{1}{\frac{4}{5} + \cfrac{1}{\frac{25}{6}s + \cfrac{1}{\frac{3}{40}}}}}$$



【例】第1章【23】(1)特別解答2

インピーダンス関数をCauer形で実現せよ。

$$Z_1(s) = \frac{(s+3)(s+5)}{(s+2)(s+4)} = \frac{s^2+8s+15}{s^2+6s+8}$$

$$\begin{array}{r|l} & \quad \frac{15}{8} \\ 8+6s+s^2 & \overline{15+8s+s^2} \\ & 15+\frac{90}{8}s+\frac{15}{8}s^2 \\ \hline & \qquad -\frac{26}{8}s-\frac{7}{8}s^2 \end{array}$$

実現できない

$$Z_1(s) = \frac{1}{\frac{s^2+6s+8}{s^2+8s+15}} = \frac{1}{\frac{8}{15}+\frac{\frac{26}{15}s+\frac{7}{15}s^2}{s^2+8s+15}}$$

$$\begin{array}{r|l} & \quad \frac{8}{15} \\ 15+8s+s^2 & \overline{8+6s+s^2} \\ & 8+\frac{64}{15}s+\frac{8}{15}s^2 \\ \hline & \qquad \frac{26}{15}s+\frac{7}{15}s^2 \end{array}$$

$$= \frac{1}{\frac{8}{15} + \frac{\frac{26}{15}s + \frac{7}{15}s^2}{s^2+8s+15}} = \frac{1}{\frac{8}{15} + \frac{1}{\frac{s^2+8s+15}{\frac{26}{15}s+\frac{7}{15}s^2}}}$$

$$= \frac{1}{\frac{8}{15} + \frac{1}{\boxed{\frac{15^2}{26s}} + \frac{\boxed{\frac{103}{26}s+s^2}}{\frac{26}{15}s+\frac{7}{15}s^2}}}$$

$$= \frac{1}{\frac{8}{15} + \frac{1}{\frac{15^2}{26s} + \frac{1}{\frac{\frac{26}{15}s+\frac{7}{15}s^2}{\frac{103}{26}s+s^2}}}}$$

$$= \frac{1}{\frac{8}{15} + \frac{1}{\frac{15^2}{26s} + \frac{1}{\frac{26^2}{103\times 15} + \frac{\frac{45}{103\times 15}s^2}{\frac{103}{26}s+s^2}}}}$$

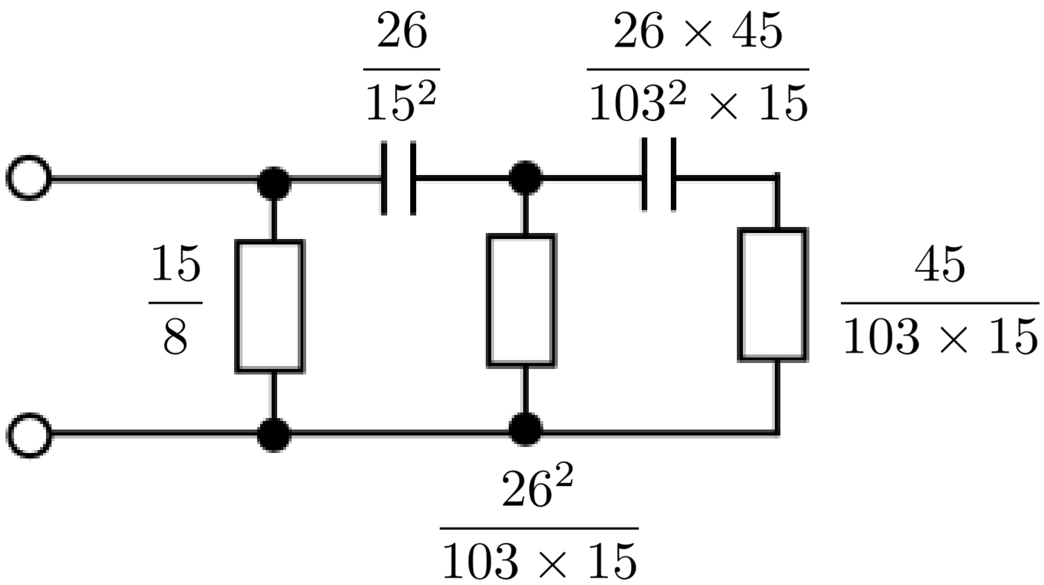
$$= \frac{1}{\frac{8}{15} + \frac{1}{\frac{15^2}{26s} + \frac{1}{\frac{26^2}{103\times 15} + \frac{1}{\frac{\frac{103}{26}s+s^2}{\frac{45}{103\times 15}s^2}}}}}$$

$$\begin{array}{r|l}
 & \boxed{\dfrac{15^2}{26s}} \\
\hline
\dfrac{26}{15}s + \dfrac{7}{15}s^2 & 15 + 8s + s^2 \\
 & 15 + \dfrac{105}{26}s \\
\hline
 & \boxed{\dfrac{103}{26}s + s^2}
\end{array}$$

$$\begin{array}{r|l}
 & \boxed{\dfrac{26^2}{103\times 15}} \\
\hline
\dfrac{103}{26}s + s^2 & \dfrac{26}{15}s + \dfrac{7}{15}s^2 \\
 & \dfrac{26}{15}s + \dfrac{26^2}{103\times 15}s^2 \\
\hline
 & \boxed{\dfrac{45}{103\times 15}s^2}
\end{array}$$

$$= \cfrac{1}{\cfrac{8}{15} + \cfrac{1}{\cfrac{15^2}{26s} + \cfrac{1}{\cfrac{26^2}{103\times 15} + \cfrac{1}{\cfrac{\frac{103}{26}s + s^2}{\frac{45}{103\times 15}s^2}}}}}$$

$$= \cfrac{1}{\cfrac{8}{15} + \cfrac{1}{\cfrac{15^2}{26s} + \cfrac{1}{\cfrac{26^2}{103\times 15} + \cfrac{1}{\cfrac{103^2\times 15}{26\times 45s} + \cfrac{103\times 15}{45}}}}}$$



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