

第2章：2端子対回路

2.7 映像パラメータ
2.11 (Bartlett の)2等分定理

キーワード：映像パラメータ, 2等分定理

学習目標：映像パラメータによる表示ができるようになる。
二等分定理を使えるようになる。

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9 四端子網

9.6 映像パラメータ

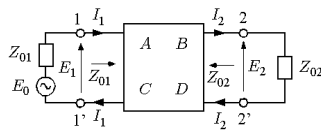
Z_{01}, Z_{02} : 映像インピーダンス

$$\begin{bmatrix} E_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} E_2 \\ I_2 \end{bmatrix}$$

伝達関数

$$\epsilon^\theta = \sqrt{\frac{E_1 I_1}{E_2 I_2}} = \sqrt{\frac{Z_{02} E_1}{Z_{01} E_2}}$$

$$Z_{01} = \sqrt{\frac{AB}{CD}}, \quad Z_{02} = \sqrt{\frac{DB}{CA}}, \quad \theta = \ln \left(\sqrt{AD} + \sqrt{BC} \right)$$



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A = D 対称回路

$$Z_{01} = Z_{02} = \sqrt{\frac{B}{C}}, \quad \theta = \ln(A + \sqrt{BC})$$

$$A = \sqrt{\frac{Z_{01}}{Z_{02}}} \cosh \theta, \quad B = \sqrt{Z_{01} Z_{02}} \sinh \theta$$

$$C = \frac{1}{\sqrt{Z_{01} Z_{02}}} \sinh \theta, \quad D = \sqrt{\frac{Z_{02}}{Z_{01}}} \cosh \theta$$

$Z_{01} = Z_{02}$ より

$$A = D = \cosh \theta, \quad B = Z_{01} \sinh \theta, \quad C = \frac{1}{Z_{01}} \sinh \theta$$

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インピーダンスパラメータ, アドミタンスパラメータで表現

$$Z_{01} = \sqrt{\frac{AB}{CD}} = \sqrt{\frac{Z_{11}}{Y_{11}}} = \sqrt{Z_{1f} Z_{1s}}$$

$$Z_{01} = \sqrt{\frac{DB}{CA}} = \sqrt{\frac{Z_{22}}{Y_{22}}} = \sqrt{Z_{2f} Z_{2s}}$$

$$\cosh \theta = \sqrt{AD} = \sqrt{Z_{11} Y_{11}} = \sqrt{Z_{22} Y_{22}} = \sqrt{\frac{Z_{1f}}{Z_{1s}}} = \sqrt{\frac{Z_{2f}}{Z_{2s}}}$$

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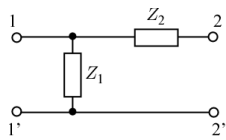
【例】第2章【27】

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_1} & 1 \end{bmatrix} \begin{bmatrix} 1 & Z_2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & Z_2 \\ \frac{1}{Z_1} & 1 + \frac{Z_2}{Z_1} \end{bmatrix}$$

$$Z_{01} = \sqrt{\frac{AB}{CD}} = \sqrt{\frac{Z_2}{\frac{1}{Z_1} \left(1 + \frac{Z_2}{Z_1}\right)}} = Z_1 \sqrt{\frac{Z_2}{Z_1 + Z_2}}$$

$$Z_{02} = \sqrt{\frac{DB}{CA}} = \sqrt{\frac{\left(1 + \frac{Z_2}{Z_1}\right) Z_2}{\frac{1}{Z_1}}} = \sqrt{(Z_1 + Z_2) Z_2}$$

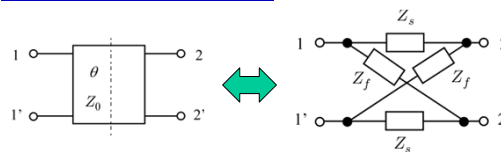
$$\theta = \cosh^{-1} \sqrt{AD} = \cosh^{-1} \sqrt{1 + \frac{Z_2}{Z_1}}$$



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9 四端子網

9.6 (Bartlett の)2等分定理



ラチス回路

Z_f : 右半分を開放して1-1' からみた回路インピーダンス

Z_s : 右半分を短絡して1-1' からみた回路インピーダンス

二等分定理

四端子網を対称軸で二等分し, その半分の回路について, Z_s, Z_f を求め, 等価なラチス回路に変換できる。

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映像パラメータとの関係

$$Z_f = Z_0 \coth \frac{\theta}{2} = Z_0 \frac{\sinh \theta}{\cosh \theta - 1}$$

$$Z_s = Z_0 \tanh \frac{\theta}{2} = Z_0 \frac{\sinh \theta}{1 + \cosh \theta}$$

$$Z_0 = \sqrt{Z_{01} Z_{02}}$$

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【例】

(1) 回路を対称に分ける

(2) 右半分を開放して1-1' からみた回路インピーダンス
 $Z_f = Z_1$

(3) 右半分を短絡して1-1' からみた回路インピーダンス
 $Z_s = \frac{Z_1 Z_2}{Z_1 + Z_2}$

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【例】第2章【49】

(1) 回路を対称に分ける

(2) 右半分を開放して1-1' からみた回路インピーダンス
 $Z_f = Z_2 + Z_3$

(3) 右半分を短絡して1-1' からみた回路インピーダンス
 $Z_s = \frac{Z_1 Z_2}{Z_1 + Z_2}$

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付録

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(証明) 映像パラメータ

Z_{01}, Z_{02} : 映像インピーダンス

$$\begin{bmatrix} E_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} E_2 \\ I_2 \end{bmatrix}$$

$$Z_{01} = \frac{E_1}{I_1} = \frac{AE_2 + BI_2}{CE_2 + DI_2}$$

$E_2 = Z_{02} I_2$ を代入

$$Z_{01} = \frac{E_1}{I_1} = \frac{AZ_{02} I_2 + BI_2}{CZ_{02} I_2 + DI_2} = \frac{(AZ_{02} + B)I_2}{(CZ_{02} + D)I_2} = \frac{AZ_{02} + B}{CZ_{02} + D} \quad (a)$$

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$$\begin{bmatrix} E_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} D & B \\ C & A \end{bmatrix} \begin{bmatrix} E_1 \\ I_1 \end{bmatrix}$$

$$Z_{02} = \frac{E_2}{I_2} = \frac{DE_1 + BI_1}{CE_1 + AI_1}$$

$E_1 = Z_{01} I_1$ を代入

$$Z_{02} = \frac{E_2}{I_2} = \frac{DZ_{01} I_1 + BI_1}{CZ_{01} I_1 + AI_1} = \frac{(DZ_{01} + B)I_1}{(CZ_{01} + D)I_1} = \frac{DZ_{01} + B}{CZ_{01} + D} \quad (b)$$

(a) より

$$Z_{01} = \frac{AZ_{02} + B}{CZ_{02} + D} \Rightarrow Z_{01}(CZ_{02} + D) = AZ_{02} + B$$

$$\Rightarrow CZ_{01}Z_{02} + DZ_{01} = AZ_{02} + B \quad (1)$$

(b) より

$$Z_{02} = \frac{DZ_{01} + B}{CZ_{01} + A} \Rightarrow Z_{02}(CZ_{01} + A) = DZ_{01} + B$$

$$\Rightarrow CZ_{02}Z_{01} + AZ_{02} = DZ_{01} + B \quad (2)$$

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(1) + (2)

$$\begin{aligned} CZ_{01}Z_{02} + DZ_{01} &= AZ_{02} + B \\ + CZ_{02}Z_{01} + AZ_{02} &= DZ_{01} + B \\ \hline 2CZ_{02}Z_{01} + DZ_{01} + AZ_{02} &= AZ_{02} + DZ_{01} + 2B \\ 2CZ_{02}Z_{01} &= 2B \\ Z_{02}Z_{01} &= \frac{2B}{2C} = \frac{B}{C} \end{aligned}$$

(1) - (2)

$$\begin{aligned} CZ_{01}Z_{02} + DZ_{01} &= AZ_{02} + B \\ - CZ_{02}Z_{01} + AZ_{02} &= DZ_{01} + B \\ \hline DZ_{01} - AZ_{02} &= AZ_{02} - DZ_{01} \\ 2DZ_{01} &= 2AZ_{02} \\ \frac{Z_{01}}{Z_{02}} &= \frac{2A}{2D} = \frac{A}{D} \end{aligned}$$

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$$Z_{02}Z_{01} = \frac{2B}{2C} = \frac{B}{C} \quad (3)$$

$$\frac{Z_{01}}{Z_{02}} = \frac{2A}{2D} = \frac{A}{D} \quad (4)$$

(4)より $Z_{01} = \frac{A}{D}Z_{02}$ を (3) へ代入

$$\begin{aligned} Z_{02} \left(\frac{A}{D}Z_{02} \right) &= \frac{B}{C} \Rightarrow \frac{A}{D}Z_{02}^2 = \frac{B}{C} \Rightarrow Z_{02}^2 = \frac{BD}{AC} \\ &\Rightarrow Z_{02} = \pm \sqrt{\frac{BD}{AC}} \end{aligned}$$

$Z_{01} = \frac{A}{D}Z_{02}$ へ代入

$$Z_{01} = \pm \frac{A}{D} \sqrt{\frac{BD}{AC}} \Rightarrow Z_{01} = \pm \sqrt{\frac{AB}{CD}}$$

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伝達定数

$$\frac{E_1}{E_2} = e^{\theta_1} \text{ とおく}$$

$$e^{\theta_1} = \frac{AE_2 + BI_2}{E_2} = A + \frac{B}{\frac{E_2}{I_2}} = A + \frac{B}{Z_{02}}$$

$$Z_{02} = \sqrt{\frac{BD}{AC}} \text{ を代入}$$

$$e^{\theta_1} = A + B \sqrt{\frac{AC}{BD}} = \sqrt{\frac{A}{D}} (\sqrt{AD} + \sqrt{BC})$$

$$\frac{I_1}{I_2} = e^{\theta_2} \text{ とおく}$$

$$e^{\theta_2} = \frac{CE_2 + DI_2}{I_2} = \frac{C}{\frac{I_2}{E_2}} + D = CZ_{02} + D$$

$$Z_{02} = \sqrt{\frac{BD}{AC}} \text{ を代入}$$

$$e^{\theta_2} = C \sqrt{\frac{BD}{AC}} + D = \sqrt{\frac{D}{A}} (\sqrt{AD} + \sqrt{BC})$$

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$$\theta = \frac{\theta_1 + \theta_2}{2} \quad \text{伝達定数}$$

θ, Z_{01}, Z_{02} : 影像パラメータ

$$\begin{aligned} e^{\theta} &= e^{\frac{\theta_1 + \theta_2}{2}} = e^{\frac{\theta_1}{2}} \cdot e^{\frac{\theta_2}{2}} \\ &= \left(\sqrt{\frac{A}{D}} (\sqrt{AD} + \sqrt{BC}) \right)^{\frac{1}{2}} \left(\sqrt{\frac{D}{A}} (\sqrt{AD} + \sqrt{BC}) \right)^{\frac{1}{2}} \\ &= \left((\sqrt{AD} + \sqrt{BC})^2 \right)^{\frac{1}{2}} \\ &= \sqrt{AD} + \sqrt{BC} \\ \theta &= \ln (\sqrt{AD} + \sqrt{BC}) \end{aligned}$$

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$$\frac{E_1}{E_2} = e^{\theta_1}, \frac{I_1}{I_2} = e^{\theta_2} \text{ より}$$

$$e^{\theta} = e^{\frac{\theta_1 + \theta_2}{2}} = e^{\frac{\theta_1}{2}} \cdot e^{\frac{\theta_2}{2}} = \sqrt{\frac{E_1 I_1}{E_2 I_2}}$$

$$\theta = \frac{1}{2} \log \frac{E_1 I_1}{E_2 I_2}$$

$\theta = \alpha + j\beta$ とおくと

α : 減衰定数 β : 位相定数

θ, Z_{01}, Z_{02} のパラメータで表す

$$\frac{E_1}{E_2} = e^{\theta_1} = \sqrt{\frac{A}{D}} (\sqrt{AD} + \sqrt{BC}) = \sqrt{\frac{A}{D}} e^{\theta} = \sqrt{\frac{Z_{01}}{Z_{02}}} e^{\theta}$$

$$\frac{I_1}{I_2} = e^{\theta_2} = \sqrt{\frac{D}{A}} (\sqrt{AD} + \sqrt{BC}) = \sqrt{\frac{Z_{02}}{Z_{01}}} e^{\theta}$$

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双曲線関数で表す $e^{\theta} = \sqrt{AD} + \sqrt{BC}$

$$\begin{aligned} \cosh \theta &= \frac{e^{\theta} + e^{-\theta}}{2} \\ &= \frac{1}{2} \left(\sqrt{AD} + \sqrt{BC} + \frac{1}{\sqrt{AD} + \sqrt{BC}} \right) \\ &= \frac{1}{2} \left(\sqrt{AD} + \sqrt{BC} + \frac{\sqrt{AD} - \sqrt{BC}}{AD + BC} \right) \\ &= \frac{1}{2} \left(\sqrt{AD} + \sqrt{BC} + \sqrt{AD} - \sqrt{BC} \right) \\ &= \sqrt{AD} \end{aligned}$$

$$\begin{aligned} \cosh \theta &= \sqrt{AD} \\ \sinh \theta &= \sqrt{BC} & \sinh \theta &= \frac{e^{\theta} - e^{-\theta}}{2} \\ \tanh \theta &= \sqrt{\frac{BC}{AD}} & \tanh \theta &= \frac{e^{\theta} - e^{-\theta}}{e^{\theta} + e^{-\theta}} \end{aligned}$$

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A, B, C, D を Z_{01}, Z_{02}, θ で表す

$$\cosh \theta = \sqrt{AD} = \sqrt{AD} \frac{A}{A} = A \sqrt{\frac{D}{A}} = A \sqrt{\frac{Z_{02}}{Z_{01}}}$$

$$\Rightarrow A = \sqrt{\frac{Z_{01}}{Z_{02}}} \cosh \theta$$

$$\cosh \theta = \sqrt{AD} = \sqrt{AD} \frac{D}{D} = D \sqrt{\frac{A}{D}} = D \sqrt{\frac{Z_{01}}{Z_{02}}}$$

$$\Rightarrow D = \sqrt{\frac{Z_{02}}{Z_{01}}} \cosh \theta$$

$$\sinh \theta = \sqrt{BC} = \sqrt{BC} \frac{B}{B} = B \sqrt{\frac{C}{B}}$$

$$Z_{01} = \sqrt{\frac{AB}{CD}}, \quad Z_{02} = \sqrt{\frac{BD}{AC}} \quad \text{より} \quad Z_{01} Z_{02} = \frac{B}{C}$$

$$\sinh \theta = B \frac{1}{\sqrt{Z_{01} Z_{02}}} \Rightarrow B = \sqrt{Z_{01} Z_{02}} \sinh \theta$$

$$\sinh \theta = \sqrt{BC} = \sqrt{BC} \frac{C}{C} = C \sqrt{\frac{B}{C}} = C \sqrt{Z_{01} Z_{02}} \Rightarrow C = \frac{1}{\sqrt{Z_{01} Z_{02}}} \sinh \theta$$

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映像パラメータを用いた四端子網の基礎方程式

$$\begin{bmatrix} E_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} E_2 \\ I_2 \end{bmatrix}$$

$$E_1 = \sqrt{\frac{Z_{01}}{Z_{02}}} (E_2 \cosh \theta + I_2 Z_{02} \sinh \theta)$$

$$I_1 = \sqrt{\frac{Z_{02}}{Z_{01}}} \left(\frac{E_2}{Z_{02}} \sinh \theta + I_2 \cosh \theta \right)$$

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