

【問題30.1】(1)

$$\omega = \frac{2\pi}{T}$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t + \sum_{n=1}^{\infty} b_n \sin n\omega t$$

【解法1】偶関数とは知らずに求める

a_0 を求める

$$a_0 = \frac{1}{T} \int_0^T v(t) dt = \frac{1}{T} \left(\int_0^{\frac{T}{4}} A dt + \int_{\frac{3T}{4}}^T A dt \right)$$

$$= \frac{A}{T} \left([t]_0^{\frac{T}{4}} + [t]_{\frac{3T}{4}}^T \right) = \frac{A}{T} \left(\frac{T}{4} + \frac{T}{4} \right) = \frac{A}{2}$$

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a_n を求める

$$a_n = \frac{2}{T} \int_0^T v(t) \cos n\omega t dt$$

$$= \frac{2}{T} \left(\int_0^{\frac{T}{4}} A \cos n\omega t dt + \int_{\frac{3T}{4}}^T A \cos n\omega t dt \right)$$

$$= \frac{2A}{T} \left(\left[\frac{1}{n\omega} \sin n\omega t \right]_0^{\frac{T}{4}} + \left[\frac{1}{n\omega} \sin n\omega t \right]_{\frac{3T}{4}}^T \right) \quad \omega T = 2\pi \text{ より}$$

$$= \frac{2A}{n\omega T} \left(\sin n\omega \frac{T}{4} - \sin 0 + \sin n\omega T - \sin n\omega \frac{3T}{4} \right)$$

$\omega T = 2\pi$ より

$$a_n = \frac{2A}{2\pi n} \left(\sin \frac{2\pi n}{4} + \sin 2\pi n - \sin \frac{6\pi n}{4} \right)$$

$$= \frac{A}{\pi n} \left(\sin \frac{\pi n}{2} - \sin \frac{3\pi n}{2} \right) = \frac{2A}{\pi n} \sin \frac{\pi n}{2}$$

$$= \sin \frac{\pi n}{2}$$

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b_n を求める

$$b_n = \frac{2}{T} \int_0^T v(t) \sin n\omega t dt$$

$$= \frac{2}{T} \left(\int_0^{\frac{T}{4}} A \sin n\omega t dt + \int_{\frac{3T}{4}}^T A \sin n\omega t dt \right)$$

$$= \frac{2A}{T} \left(\left[-\frac{1}{n\omega} \cos n\omega t \right]_0^{\frac{T}{4}} + \left[-\frac{1}{n\omega} \cos n\omega t \right]_{\frac{3T}{4}}^T \right)$$

$$= \frac{2A}{n\omega T} \left(\cos n\omega \frac{T}{4} - \cos 0 + \cos n\omega T - \cos n\omega \frac{3T}{4} \right)$$

$\omega T = 2\pi$ より

$$b_n = \frac{2A}{2\pi n} \left(\cos \frac{2\pi n}{4} - 1 + \cos 2\pi n - \cos \frac{6\pi n}{4} \right)$$

$$= \frac{A}{\pi n} \left(\cos \frac{\pi n}{2} - \cos \frac{3\pi n}{2} \right) = 0$$

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$$a_1 = \frac{2A}{\pi} \sin \frac{\pi}{2} = \frac{2A}{\pi}$$

$$a_2 = \frac{2A}{2\pi} \sin \frac{2\pi}{2} = 0$$

$$a_3 = \frac{2A}{3\pi} \sin \frac{3\pi}{2} = -\frac{2A}{3\pi}$$

よって

$$v(t) = \frac{A}{2} + \frac{2A}{\pi} \left(1 \cdot \cos \omega t - \frac{1}{3} \cos 3\omega t + \frac{1}{5} \cos 5\omega t + \dots \right)$$

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【解法2】偶関数を使う

偶関数より $b_n = 0$

a_0 を求める

(A1)

$$a_0 = \frac{1}{T} \int_0^T v(t) dt = \frac{1}{T} \left(\int_0^{\frac{T}{4}} A dt + \int_{\frac{3T}{4}}^T A dt \right)$$

(A2) 半周期を2倍

$$a_0 = \frac{2}{T} \int_0^{\frac{T}{2}} v(t) dt = \frac{2}{T} \int_0^{\frac{T}{4}} A dt \quad \text{半周期が使える}$$

(A3) 周期を $-\frac{T}{4} \sim \frac{3T}{4}$ と考える

$$a_0 = \frac{1}{T} \int_{-\frac{T}{4}}^{\frac{3T}{4}} v(t) dt = \frac{1}{T} \int_{-\frac{T}{4}}^{\frac{T}{4}} A dt$$

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(解法1)で解く

$$a_0 = \frac{A}{T} \left([t]_0^{\frac{T}{4}} + [t]_{\frac{3T}{4}}^T \right) = \frac{A}{T} \left(\frac{T}{4} + \frac{T}{4} \right) = \frac{A}{2}$$

(解法2)で解く

$$a_0 = \frac{2A}{T} [t]_0^{\frac{T}{4}} = \frac{2A}{T} \frac{T}{4} = \frac{A}{2}$$

(解法3)で解く

$$a_0 = \frac{A}{T} [t]_{-\frac{T}{4}}^{\frac{T}{4}} = \frac{A}{T} \left(\frac{T}{4} + \frac{T}{4} \right) = \frac{A}{2}$$

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(A2) 半周期で解く * (A1) 全周期との違いを示す

$$\begin{aligned}
 a_n &= \frac{2}{T} \int_0^T v(t) \cos n\omega t \, dt \\
 &= \frac{2}{T} \left(\int_0^{\frac{T}{4}} A \cos n\omega t \, dt + \int_{\frac{3T}{4}}^T A \cos n\omega t \, dt \right) \\
 &= \frac{2A}{T} \left(\left[\frac{1}{n\omega} \sin n\omega t \right]_0^{\frac{T}{4}} + \left[\frac{1}{n\omega} \sin n\omega t \right]_{\frac{3T}{4}}^T \right) \\
 &= \frac{2A}{n\omega T} \left(\sin n\omega \frac{T}{4} - \sin 0 + \sin n\omega T - \sin n\omega \frac{3T}{4} \right)
 \end{aligned}$$

$\omega T = 2\pi$ より

$$\begin{aligned}
 a_n &= \frac{2A}{2\pi n} \left(\sin \frac{2\pi n}{4} + \sin \frac{6\pi n}{4} \right) \\
 &= \frac{2A}{\pi n} \left(\sin \frac{\pi n}{2} + \sin \frac{3\pi n}{2} \right) = \frac{2A}{\pi n} \sin \frac{\pi n}{2}
 \end{aligned}$$

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(A3) で解く * (A1) との違いを示す

$$\begin{aligned}
 a_n &= \frac{2}{T} \int_0^T v(t) \cos n\omega t \, dt \\
 &= \frac{2}{T} \left(\int_0^{\frac{T}{4}} A \cos n\omega t \, dt + \int_{\frac{3T}{4}}^T A \cos n\omega t \, dt \right) \\
 &= \frac{2A}{T} \left(\left[\frac{1}{n\omega} \sin n\omega t \right]_0^{\frac{T}{4}} + \left[\frac{1}{n\omega} \sin n\omega t \right]_{\frac{3T}{4}}^T \right) \\
 &= \frac{2A}{n\omega T} \left(\sin n\omega \frac{T}{4} - \sin 0 + \sin n\omega T - \sin n\omega \frac{3T}{4} \right) \\
 &\quad - \sin \left(-n\omega \frac{T}{4} \right) \\
 &\quad = + \sin n\omega \frac{T}{4}
 \end{aligned}$$

$\omega T = 2\pi$ より

$$\begin{aligned}
 a_n &= \frac{2A}{2\pi n} \left(\sin \frac{2\pi n}{4} + \sin \frac{6\pi n}{4} \right) \\
 &= \frac{2A}{\pi n} \left(\sin \frac{\pi n}{2} + \sin \frac{3\pi n}{2} \right) = \frac{2A}{\pi n} \sin \frac{\pi n}{2}
 \end{aligned}$$

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