

**[問題30.1] (2)**

$$\omega = \frac{2\pi}{T}$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t + \sum_{n=1}^{\infty} b_n \sin n\omega t$$

**【解法1】奇関数とは知らずに求める**

$a_0$  を求める

$$a_0 = \frac{1}{T} \int_0^T v(t) dt = \frac{1}{T} \left( \int_0^{\frac{T}{2}} \frac{2A}{T} t dt + \int_{\frac{T}{2}}^T \left( \frac{2A}{T} t - 2A \right) dt \right)$$

$$= \frac{1}{T} \left( \frac{2A}{T} \left[ \frac{1}{2} t^2 \right]_0^{\frac{T}{2}} + \frac{2A}{T} \left[ \frac{1}{2} t^2 \right]_{\frac{T}{2}}^T - 2A \left[ t \right]_{\frac{T}{2}}^T \right)$$

$$= \frac{1}{T} \left( \frac{2A}{T} \left( \frac{1}{8} T^2 + \frac{1}{2} T^2 - \frac{1}{8} T^2 \right) - 2A \frac{T}{2} \right)$$

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$$= \frac{1}{T} \left( \frac{2A}{T} \left( \frac{1}{8} T^2 + \frac{1}{2} T^2 - \frac{1}{8} T^2 \right) - 2A \frac{T}{2} \right)$$

$$= \frac{1}{T} \left( \frac{2A}{T} \frac{1}{2} T^2 - 2A \frac{T}{2} \right)$$

$$= \frac{1}{T} (AT - AT)$$

$$= 0$$

**別解**

$$a_0 = \frac{1}{T} \int_0^T v(t) dt = \frac{1}{T} \left( \int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{2A}{T} t dt \right)$$

$$= \frac{2A}{T^2} \left[ \frac{1}{2} t^2 \right]_{-\frac{T}{2}}^{\frac{T}{2}}$$

$$= 0$$

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$$a_n = \frac{2}{T} \int_0^T v(t) \cos n\omega t dt$$

$$= \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{2A}{T} t \cos n\omega t dt$$

$$= \frac{4A}{T^2} \int_{-\frac{T}{2}}^{\frac{T}{2}} t \left( \frac{1}{n\omega} \sin n\omega t \right)' dt$$

$$= \frac{4A}{T^2} \left\{ \left[ t \left( \frac{1}{n\omega} \sin n\omega t \right) \right]_{-\frac{T}{2}}^{\frac{T}{2}} - \int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{1}{n\omega} \sin n\omega t dt \right\} \quad \text{部分積分}$$

$$= \frac{4A}{n\omega T^2} \left\{ \left[ t \sin n\omega t \right]_{-\frac{T}{2}}^{\frac{T}{2}} - \int_{-\frac{T}{2}}^{\frac{T}{2}} \sin n\omega t dt \right\}$$

$$= - \left[ \frac{1}{n\omega} \cos n\omega t \right]_{-\frac{T}{2}}^{\frac{T}{2}}$$

$$= \frac{4A}{n\omega T^2} \left\{ \frac{T}{2} \sin n\omega \frac{T}{2} - 0 + \frac{1}{n\omega} \left( \cos n\omega \frac{T}{2} - \cos n\omega \left( -\frac{T}{2} \right) \right) \right\}$$

$$= 0$$

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$$= \frac{4A}{n\omega T^2} \left\{ \frac{T}{2} \sin n\omega \frac{T}{2} - 0 + \frac{1}{n\omega} \left( \cos n\omega \frac{T}{2} - \cos n\omega \left( -\frac{T}{2} \right) \right) \right\}$$

$$= \frac{4A}{n\omega T^2} \frac{T}{2} \sin n\omega \frac{T}{2} \quad = 0$$

$\omega T = 2\pi$  より

$$a_n = \frac{4A}{n\omega T^2} \frac{T}{2} \sin n\pi = 0$$

$$= 0$$

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$$b_n = \frac{2}{T} \int_0^T v(t) \sin n\omega t dt$$

$$= \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{2A}{T} t \sin n\omega t dt$$

$$= \frac{4A}{T^2} \int_{-\frac{T}{2}}^{\frac{T}{2}} t \left( -\frac{1}{n\omega} \cos n\omega t \right)' dt$$

$$= \frac{4A}{T^2} \left\{ \left[ t \left( -\frac{1}{n\omega} \cos n\omega t \right) \right]_{-\frac{T}{2}}^{\frac{T}{2}} - \int_{-\frac{T}{2}}^{\frac{T}{2}} -\frac{1}{n\omega} \cos n\omega t dt \right\}$$

$$= - \frac{4A}{n\omega T^2} \left\{ \left[ t \cos n\omega t \right]_{-\frac{T}{2}}^{\frac{T}{2}} - \int_{-\frac{T}{2}}^{\frac{T}{2}} \cos n\omega t dt \right\}$$

$$= \left[ \frac{1}{n\omega} \sin n\omega t \right]_{-\frac{T}{2}}^{\frac{T}{2}}$$

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$$= - \frac{4A}{n\omega T^2} \left\{ \frac{T}{2} \cos n\omega \frac{T}{2} - \left( -\frac{T}{2} \right) \cos \left( -n\omega \frac{T}{2} \right) - \frac{1}{n\omega} \left( \sin n\omega \frac{T}{2} - \sin \left( -n\omega \frac{T}{2} \right) \right) \right\}$$

$$= - \frac{4A}{n\omega T^2} \left\{ 2 \frac{T}{2} \cos n\omega \frac{T}{2} - \frac{2}{n\omega} \sin n\omega \frac{T}{2} \right\}$$

$$= - \frac{8A}{n\omega T^2} \left\{ \frac{T}{2} \cos n\omega \frac{T}{2} - \frac{1}{n\omega} \sin n\omega \frac{T}{2} \right\}$$

$\omega T = 2\pi$  より

$$b_n = - \frac{8A}{n2\pi T} \left\{ \frac{T}{2} \cos n\pi - \frac{1}{n\omega} (\sin n\pi) \right\}$$

$$= - \frac{2A}{n\pi} \cos n\pi \quad = 0$$

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$$b_1 = -\frac{2A}{\pi} \cos \pi = \frac{2A}{\pi}$$

$$b_2 = -\frac{2A}{2\pi} \cos 2\pi = -\frac{2A}{2\pi}$$

$$b_3 = -\frac{2A}{3\pi} \cos 3\pi = \frac{2A}{3\pi}$$

よって

$$v(t) = \frac{2A}{\pi} \left( \sin \omega t - \frac{1}{2} \sin 2\omega t + \frac{1}{3} \sin 3\omega t + \dots \right)$$

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**【解法2】奇関数を使う**

奇関数より  $a_0 = 0, a_n = 0$

$b_n$  を求める

(A1)

$$b_n = \frac{2}{T} \int_0^T v(t) \sin n\omega t dt$$

$$= \frac{2}{T} \left( \int_0^{\frac{T}{2}} \frac{2A}{T} t \sin n\omega t dt + \int_{\frac{T}{2}}^T \left( \frac{2A}{T} t - 2A \right) \sin n\omega t dt \right)$$

(A2) 半周期を2倍

$$b_n = \frac{4}{T} \int_0^{\frac{T}{2}} v(t) \sin n\omega t dt = \frac{4}{T} \int_0^{\frac{T}{2}} \frac{2A}{T} t \sin n\omega t dt$$

(A3) 周期を  $-\frac{T}{2} \sim \frac{T}{2}$  と考える

$$b_n = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} v(t) \sin n\omega t dt = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{2A}{T} t \sin n\omega t dt$$

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$b_n$  を求める

(A2) で解く

$$b_n = \frac{4}{T} \int_0^{\frac{T}{2}} \frac{2A}{T} t \sin n\omega t dt$$

$$= \frac{8A}{T^2} \int_0^{\frac{T}{2}} t \left( -\frac{1}{n\omega} \cos n\omega t \right)' dt$$

$$= \frac{8A}{T^2} \int_0^{\frac{T}{2}} \frac{1}{n\omega} \sin n\omega t dt$$

部分積分を用いる

$$b_n = \frac{8A}{T^2} \left\{ \left[ t \left( -\frac{1}{n\omega} \cos n\omega t \right) \right]_0^{\frac{T}{2}} - \int_0^{\frac{T}{2}} -\frac{1}{n\omega} \cos n\omega t dt \right\}$$

$$= -\frac{8A}{n\omega T^2} \left\{ \left[ t \cos n\omega t \right]_0^{\frac{T}{2}} - \int_0^{\frac{T}{2}} \cos n\omega t dt \right\}$$

$$= \frac{8A}{n\omega T^2} \left[ \frac{1}{n\omega} \sin n\omega t \right]_0^{\frac{T}{2}}$$

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$$= -\frac{8A}{n\omega T^2} \left\{ \frac{T}{2} \cos n\omega \frac{T}{2} - 0 - \frac{1}{n\omega} \left( \sin n\omega \frac{T}{2} - \sin 0 \right) \right\}$$

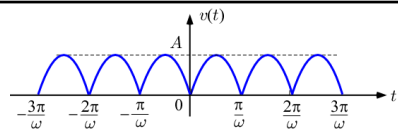
$\omega T = 2\pi$  より

$$b_n = -\frac{8A}{n2\pi T} \left\{ \frac{T}{2} \cos n\pi - \frac{1}{n\omega} (\sin n\pi) \right\}$$

$$= -\frac{2A}{n\pi} \cos n\pi$$

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[問題30.1] (3)

$$\omega = \frac{2\pi}{T}$$


$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t + \sum_{n=1}^{\infty} b_n \sin n\omega t$$

偶関数より  $b_n = 0$

$a_0$  を求める

(解法1)

$$a_0 = \frac{1}{T} \int_0^T v(t) dt = \frac{1}{T} \left( \int_0^{\frac{T}{2}} A \sin \omega t dt + \int_{\frac{T}{2}}^T (-A \sin \omega t) dt \right)$$

(解法2) 半周期を2倍

$$a_0 = \frac{2}{T} \int_0^{\frac{T}{2}} v(t) dt = \frac{2}{T} \int_0^{\frac{T}{2}} A \sin \omega t dt$$

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(解法1) で解く

$$a_0 = \frac{1}{T} \left( \int_0^{\frac{T}{2}} A \sin \omega t dt + \int_{\frac{T}{2}}^T (-A \sin \omega t) dt \right)$$

$$= \frac{A}{T} \left( \left[ -\frac{1}{\omega} \cos \omega t \right]_0^{\frac{T}{2}} + \left[ \frac{1}{\omega} \cos \omega t \right]_{\frac{T}{2}}^T \right)$$

$$= \frac{A}{\omega T} \left( [-\cos \omega t]_0^{\frac{T}{2}} + [\cos \omega t]_{\frac{T}{2}}^T \right)$$

$$= \frac{A}{\omega T} \left( -\cos \frac{\omega T}{2} + \cos 0 + \cos \omega T - \cos \frac{\omega T}{2} \right)$$

$$= \frac{A}{2\pi} (-\cos \pi + 1 + \cos 2\pi - \cos \pi)$$

$$= \frac{A}{2\pi} (1 + 1 + 1 + 1)$$

$$= \frac{2A}{\pi}$$

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(解法2)で解く

$$\begin{aligned}
 a_0 &= \frac{2}{T} \int_0^{\frac{T}{2}} A \sin \omega t \\
 &= \frac{2A}{T} \left( \left[ -\frac{1}{\omega} \cos \omega t \right]_0^{\frac{T}{2}} \right) \\
 &= \frac{2A}{\omega T} \left( [-\cos \omega t]_0^{\frac{T}{2}} \right) \\
 &= \frac{2A}{\omega T} \left( -\cos \frac{\omega T}{2} + \cos 0 \right) \\
 &= \frac{2A}{2\pi} (-\cos \pi + 1) \\
 &= \frac{2A}{2\pi} (1 + 1) \\
 &= \frac{2A}{\pi}
 \end{aligned}$$

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(解法2)で解く

$$\begin{aligned}
 a_n &= \frac{4}{T} \int_0^{\frac{T}{2}} v(t) \cos n\omega t \, dt \\
 &= \frac{4}{T} \int_0^{\frac{T}{2}} A \sin \omega t \cos n\omega t \, dt
 \end{aligned}$$

$2 \cos \alpha \sin \beta = \sin(\alpha + \beta) - \sin(\alpha - \beta)$  を用いて

$$\begin{aligned}
 a_n &= \frac{2A}{T} \int_0^{\frac{T}{2}} (\sin(n+1)\omega t - \sin(n-1)\omega t) \, dt \\
 &= \frac{2A}{T} \left( \frac{1}{(n+1)\omega} [-\cos(n+1)\omega t]_0^{\frac{T}{2}} - \frac{1}{(n-1)\omega} [-\cos(n-1)\omega t]_0^{\frac{T}{2}} \right) \\
 &= \frac{2A}{\omega T} \left( -\frac{1}{n+1} [\cos(n+1)\omega t]_0^{\frac{T}{2}} + \frac{1}{n-1} [\cos(n-1)\omega t]_0^{\frac{T}{2}} \right) \\
 &= \frac{2A}{\omega T} \left\{ -\frac{1}{n+1} \left( \cos(n+1) \frac{\omega T}{2} - \cos 0 \right) + \frac{1}{n-1} \left( \cos(n-1) \frac{\omega T}{2} - \cos 0 \right) \right\}
 \end{aligned}$$

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$\omega = \frac{2\pi}{T}$  を用いて

$$\begin{aligned}
 a_n &= \frac{2A}{2\pi} \left\{ -\frac{1}{n+1} (\cos(n+1)\pi - 1) + \frac{1}{n-1} (\cos(n-1)\pi - 1) \right\} \\
 a_1 &= \frac{A}{\pi} \left\{ -\frac{1}{2} (\cos 2\pi - 1) + \frac{1}{1-1} (\cos 0 - 1) \right\} = 0 \\
 a_2 &= \frac{A}{\pi} \left\{ -\frac{1}{3} (\cos 3\pi - 1) + \frac{1}{1} (\cos \pi - 1) \right\} = \frac{A}{\pi} \left\{ -\frac{1}{3} (-2) - 2 \right\} = \frac{A}{\pi} \left( -\frac{4}{3} \right) \\
 a_3 &= \frac{A}{\pi} \left\{ -\frac{1}{4} (\cos 4\pi - 1) + \frac{1}{2} (\cos 2\pi - 1) \right\} = 0 \\
 a_4 &= \frac{A}{\pi} \left\{ -\frac{1}{5} (\cos 5\pi - 1) + \frac{1}{3} (\cos 3\pi - 1) \right\} = \frac{A}{\pi} \left\{ -\frac{1}{5} (-2) + \frac{1}{3} (-2) \right\} \\
 &= \frac{A}{\pi} \left( \frac{2}{5} - \frac{2}{3} \right) = \frac{A}{\pi} \left( \frac{6-10}{15} \right) = \frac{A}{\pi} \left( -\frac{4}{15} \right)
 \end{aligned}$$

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よって

$$v(t) = \frac{4A}{\pi} \left( \frac{1}{2} - \frac{1}{3} \cos 2\omega t - \frac{1}{15} \cos 4\omega t \dots \right)$$

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