

第 24 章 : 非正弦波交流

24.4 ひずみ波の実効値

24.5 ひずみ率

キーワード : ひずみ波の実効値, ひずみ率

学習目標 : ひずみ波の実効値, ひずみ率を計算できる。
ひずみ波の交流電力を求めることができる。

6 ひずみ波

6.4 ひずみ波の実効値

$$i(t) = I_0 + I_{m1} \sin(\omega t + \theta_1) + I_{m2} \sin(2\omega t + \theta_2) + \dots$$

$$|I| = \sqrt{\frac{1}{T} \int_0^T i^2(t) dt}$$

$$\omega = \frac{2\pi}{T}$$

$$|I| = \sqrt{I_0^2 + |I_1|^2 + |I_2|^2 + \dots}$$

証明

$$\begin{aligned} i(t)^2 &= (I_0 + I_{m1} \sin(\omega t + \theta_1) + I_{m2} \sin(2\omega t + \theta_2) + \dots) \\ &\quad \times (I_0 + I_{m1} \sin(\omega t + \theta_1) + I_{m2} \sin(2\omega t + \theta_2) + \dots) \\ &= (I_0^2 + I_{m1}^2 \sin^2(\omega t + \theta_1) + I_{m2}^2 \sin^2(2\omega t + \theta_2) + \dots) \\ &\quad + 2I_0 (I_{m1} \sin(\omega t + \theta_1) + I_{m2} \sin(2\omega t + \theta_2) + \dots) \\ &\quad + 2I_{m1} \sin(\omega t + \theta_1) (I_{m2} \sin(2\omega t + \theta_2) + \dots) \end{aligned}$$

$$\frac{1}{T} \int_0^T I_0^2 dt = I_0^2$$

$$\begin{aligned} \cos 2\omega t &= \cos^2 \omega t - \sin^2 \omega t \\ &= 1 - 2 \sin^2 \omega t \end{aligned}$$

$$\frac{1}{T} \int_0^T I_{mn}^2 \sin^2(n\omega t + \theta_n) dt$$

$$\Rightarrow \sin^2 \omega t = \frac{1 - \cos 2\omega t}{2}$$

$$= \frac{I_{mn}^2}{2T} \int_0^T (1 - \cos 2(n\omega t + \theta_n)) dt = \left(\frac{I_{mn}}{\sqrt{2}} \right)^2 = |I_n|^2$$

$$\frac{1}{T} \int_0^T I_0 I_{mn} \sin(n\omega t + \theta_n) dt = 0$$

$$\frac{1}{T} \int_0^T I_{mh} I_{mk} \sin(h\omega t + \theta_h) \sin(k\omega t + \theta_k) dt = 0 \quad (h \neq k)$$

$$\begin{aligned}
& \frac{1}{T} \int_0^T I_{mh} I_{mk} \sin(h\omega t + \theta_h) \sin(k\omega t + \theta_k) dt \\
&= -\frac{I_{mh} I_{mk}}{2T} \int_0^T \cos((h+k)\omega t + (\theta_h + \theta_k)) - \cos((h-k)\omega t + (\theta_h - \theta_k)) dt \\
&= -\frac{I_{mh} I_{mk}}{2T} \left\{ \frac{1}{h+k} [\sin((h+k)\omega t + (\theta_h + \theta_k))]_0^T \right. \\
&\quad \left. - \frac{1}{h+k} [\sin((h-k)\omega t + (\theta_h - \theta_k))]_0^T \right\} \\
&= -\frac{I_{mh} I_{mk}}{2T} \left[\frac{1}{h+k} \{ \sin(\underbrace{(h+k)\omega T + (\theta_h + \theta_k)}_{= 2\pi}) - \sin(\theta_h + \theta_k) \} \right. \\
&\quad \left. - \frac{1}{h+k} \{ \sin(\underbrace{(h-k)\omega T + (\theta_h - \theta_k)}_{= 2\pi}) - \sin(\theta_h - \theta_k) \} \right] \\
&= -\frac{I_{mh} I_{mk}}{2T} \left[\frac{1}{h+k} \{ \sin(\underbrace{(h+k)2\pi + (\theta_h + \theta_k)}_{= \sin(\theta_h + \theta_k)}) - \sin(\theta_h + \theta_k) \} \right. \\
&\quad \left. - \frac{1}{h+k} \{ \sin(\underbrace{(h-k)2\pi + (\theta_h - \theta_k)}_{= \sin(\theta_h - \theta_k)}) - \sin(\theta_h - \theta_k) \} \right] \\
&= 0
\end{aligned}$$

$$\begin{aligned}
\frac{1}{T} \int_0^T i(t)^2 dt &= \frac{1}{T} \int_0^T (I_0^2 + I_{m1}^2 \sin^2(\omega t + \theta_1) + I_{m2}^2 \sin^2(2\omega t + \theta_2) + \dots) \\
&\quad + 2I_0 (I_{m1} \sin(\omega t + \theta_1) + I_{m2} \sin(2\omega t + \theta_2) + \dots) \\
&\quad + 2I_{m1} \sin(\omega t + \theta_1) (I_{m2} \sin(2\omega t + \theta_1) + \dots) \\
&= I_0^2 + \frac{I_{m1}^2}{2} + \frac{I_{m2}^2}{2} + \dots \\
&= I_0^2 + \underbrace{\left(\frac{I_{m1}}{\sqrt{2}}\right)^2}_{= I_1^2} + \underbrace{\left(\frac{I_{m2}}{\sqrt{2}}\right)^2}_{= I_2^2} + \dots
\end{aligned}$$

よって

$$|I| = \sqrt{I_0^2 + |I_1|^2 + |I_2|^2 + \dots}$$

[例題 31.2]

$$\begin{aligned}v(t) &= \underbrace{4\sqrt{2}}_{=\sqrt{2}|V_1|} \sin(t - 30^\circ) + \underbrace{2\sqrt{2}}_{=\sqrt{2}|V_2|} \sin(2t + 30^\circ) \\ &\quad + \underbrace{2\sqrt{2}}_{=\sqrt{2}|V_3|} \sin(3t - 30^\circ) + \underbrace{\sqrt{2}}_{=\sqrt{2}|V_4|} \sin(4t - 60^\circ)\end{aligned}$$

$$|V_1| = 4$$

$$|V_2| = 2$$

$$|V_3| = 2$$

$$|V_4| = 1$$

よって

$$\begin{aligned}|V| &= \sqrt{|V_1|^2 + |V_2|^2 + |V_3|^2 + |V_4|^2} \\ &= \sqrt{4^2 + 2^2 + 2^2 + 1^2} = 5\end{aligned}$$

24 非正弦波交流

24.5 波形率・波高率・ひずみ率

$$i(t) = I_0 + I_{m1} \sin(\omega t - \theta_1) + I_{m2} \sin(2\omega t - \theta_2) + \dots$$

$$I_1 = \frac{I_{m1}}{\sqrt{2}}, \quad I_2 = \frac{I_{m2}}{\sqrt{2}}$$

$$\text{波形率} = \frac{\text{実効値}}{\text{平均値}} = \frac{|I|}{I_a} = \frac{\sqrt{\frac{1}{T} \int_0^T i^2 dt}}{\frac{1}{T} \int_0^T |i| dt} = \frac{\sqrt{I_0^2 + I_1^2 + \dots}}{\frac{1}{T} \int_0^T |i| dt}$$

$$\text{波高率} = \frac{\text{最大値}}{\text{実効値}} = \frac{I_m}{|I|} = \frac{I_m}{\sqrt{\frac{1}{T} \int_0^T i^2 dt}} = \frac{I_m}{\sqrt{I_0^2 + I_1^2 + \dots}}$$

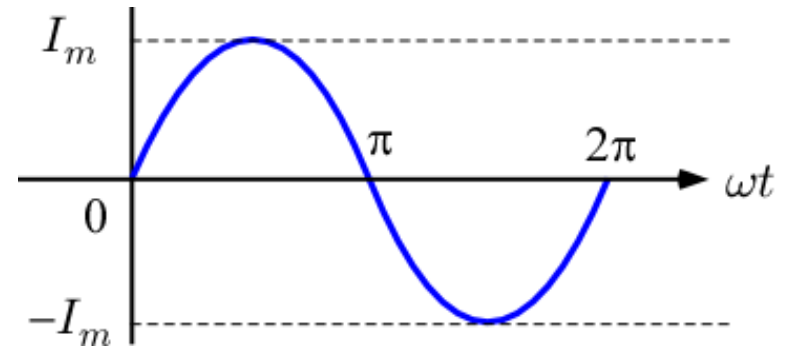
$$\text{ひずみ率} = \frac{\text{全高調波の実効値}}{\text{基本波の実効値}} = \frac{\sqrt{|I_2|^2 + |I_3|^2 + \dots}}{|I_1|}$$

正弦波 $i(t) = I_m \sin \omega t$

$$\text{波形率} = \frac{\frac{I_m}{\sqrt{2}}}{\frac{2}{\pi} I_m} = \frac{\pi}{2\sqrt{2}} = 1.11$$

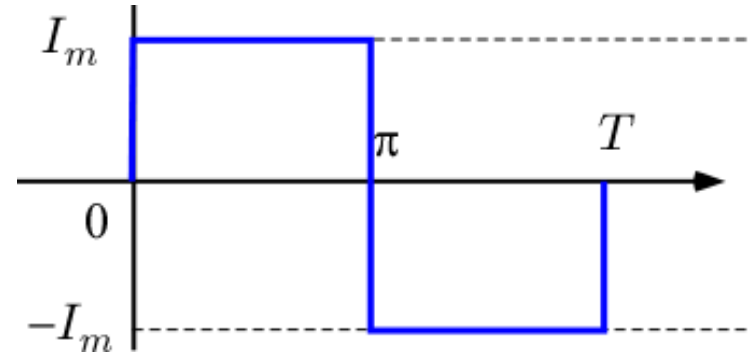
$$\text{波高率} = \frac{I_m}{\frac{I_m}{\sqrt{2}}} = \sqrt{2} = 1.41$$

$$\text{ひずみ率} = \frac{0}{\frac{I_m}{\sqrt{2}}} = 0 \quad 2\omega t, 3\omega t, \dots \text{はない}$$



方形波

$$i(t) = \begin{cases} I_m & (0 < t < \frac{T}{2}) \\ -I_m & (\frac{T}{2} < t < T) \end{cases}$$



$$\text{波形率} = \frac{I_m}{I_m} = 1$$

$$\text{波高率} = \frac{I_m}{I_m} = 1$$

$$i(t) = \frac{8}{T} I_m \left(\sin t + \frac{1}{3} \sin 3t + \frac{1}{5} \sin 5t + \dots \right)$$

$$\text{ひずみ率} = \frac{\sqrt{\left(\frac{1}{3\sqrt{2}}\right)^2 + \left(\frac{1}{5\sqrt{2}}\right)^2 + \dots}}{\frac{1}{\sqrt{2}}} = \frac{\sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{5}\right)^2 + \dots}}{1}$$

$$= \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{5}\right)^2 + \dots}$$

[例題 31.2]

$$\begin{aligned} v(t) &= \underbrace{4\sqrt{2}}_{=\sqrt{2}|V_1|} \sin(t - 30^\circ) + \underbrace{2\sqrt{2}}_{=\sqrt{2}|V_2|} \sin(2t + 30^\circ) \\ &\quad + \underbrace{2\sqrt{2}}_{=\sqrt{2}|V_3|} \sin(3t - 30^\circ) + \underbrace{\sqrt{2}}_{=\sqrt{2}|V_4|} \sin(4t - 60^\circ) \end{aligned}$$

ひずみ率

$$\begin{aligned} k &= \frac{\sqrt{|V_2|^2 + |V_3|^2 + |V_4|^2}}{|V_1|} \\ &= \frac{\sqrt{2^2 + 2^2 + 1^2}}{4} = \frac{3}{4} \end{aligned}$$

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