第 24 章 : 非正弦波交流

24.6 ひずみ波交流電力

キーワード : ひずみ波交流電力

学習目標 : ひずみ波の交流電力を求めることができる。

24 非正弦波交流

24.6 ひずみ波交流電力

【復習】(単相)交流電力

$$e(t) = E_m \sin(\omega t + \theta_n)$$

$$i(t) = I_m \sin(\omega t + \theta_n - \varphi)$$

$$|E| = \frac{E_m}{\sqrt{2}}$$

$$|I| = \frac{I_m}{\sqrt{2}}$$

瞬時電力 $p = i \cdot e$ [W]

有効電力 $P = \frac{1}{T}\int_0^T p\,dt = |I||E|\cos\varphi$ [W]

無効電力 $P_r = |I||E|\sin\varphi$ [Var]

皮相電力 $P_a = |I||E|$ [VA]

力率 $\cos\varphi = \frac{P}{P_a}$

ひずみ波交流電力

$$e = E_0 + \sum_{n=1}^{\infty} E_{mn} \sin(n\omega t + \theta_n)$$

$$i = I_0 + \sum_{n=1}^{\infty} I_{mn} \sin(n\omega t + \theta_n - \varphi_n)$$

瞬時電力

$$p = i \cdot e$$

$$\begin{aligned}
&= E_0 I_0 + \sum_{n=1}^{\infty} E_{mn} I_{mn} \sin(n\omega t + \theta_n) \sin(n\omega t + \theta_n - \varphi_n) \\
&\quad + \sum_{n=1}^{\infty} \{E_0 I_{mn} \sin(n\omega t + \theta_n - \varphi_n) + I_0 E_{mn} \sin(n\omega t + \theta_n)\} \\
&\quad + \sum_{\substack{n=1 \\ n \neq k}}^{\infty} \sum_{k=1}^{\infty} E_{mn} I_{mk} \sin(n\omega t + \theta_n) \sin(k\omega t + \theta_k - \varphi_k)
\end{aligned}$$

有効電力

$$P = \frac{1}{T}\int_0^T p\ dt$$

$$= \underbrace{\frac{1}{T}\int_0^T E_0 I_0 dt}_{= E_0 I_0} + \underbrace{\frac{1}{T}\int_0^T \sum_{n=1}^{\infty} E_{mn} I_{mn} \sin(n\omega t + \theta_n)\sin(n\omega t + \theta_n - \varphi_n) dt}_{= \sum_{n=1}^{\infty} \frac{E_{mn} I_{mn} \cos\varphi_n}{2}}$$

$$+ \underbrace{\sum_{n=1}^{\infty} \{E_0 I_{mn} \sin(n\omega t + \theta_n - \varphi_n) + I_0 E_{mn} \sin(n\omega t + \theta_n)\}}_{= 0}$$

$$+ \underbrace{\sum_{\substack{n=1 \\ n \neq k}}^{\infty} \sum_{k=1}^{\infty} E_{mn} I_{mk} \sin(n\omega t + \theta_n)\sin(k\omega t + \theta_k - \varphi_k)}_{= 0}$$

$$= E_0 I_0 + \sum_{n=1}^{\infty} \frac{E_{mn}}{\sqrt{2}} \frac{I_{mn}}{\sqrt{2}} \cos\varphi_n$$

$$\frac{1}{T}\int_0^T \sum_{n=1}^{\infty} E_{mn} I_{mn} \sin(n\omega t + \theta_n) \sin(n\omega t + \theta_n - \varphi_n) dt = \sum_{n=1}^{\infty} \frac{E_{mn} I_{mn} \cos\varphi_n}{2}$$ の証明

$$\sin(\alpha)\sin(\beta) = -\frac{1}{2}\left\{\cos(\alpha+\beta) - \cos(\alpha-\beta)\right\}$$ を用いて

$$-\frac{E_{mn} I_{mn}}{2T} \sum_{n=1}^{\infty} \int_0^T \left(\cos(2n\omega t + 2\theta_n - \varphi_n) - \cos(\varphi_n)\right) dt$$

$$= -\frac{E_{mn} I_{mn}}{2T} \sum_{n=1}^{\infty} \left[\frac{1}{2n\omega} \sin(2n\omega t + 2\theta_n - \varphi_n) - \cos(\varphi_n) t\right]_0^T$$

$$= -\frac{E_{mn} I_{mn}}{2T} \sum_{n=1}^{\infty} \left(\underline{\frac{1}{2n\omega} \sin(2n\omega T + 2\theta_n - \varphi_n)} - \cos(\varphi_n) T - \underline{\frac{1}{2n\omega} \sin(2\theta_n - \varphi_n)}\right)$$

$$= \frac{1}{2n\omega} \sin(2\theta_n - \varphi_n)$$

$$= \sum_{n=1}^{\infty} \frac{E_{mn} I_{mn}}{2} \cos(\varphi_n)$$

有効電力

$$P = E_0 I_0 + \sum_{n=1}^{\infty} |E_n||I_n| \cos\varphi_n$$

$$= E_0 I_0 + |E_1||I_1| \cos\varphi_1 + |E_2||I_2| \cos\varphi_2 + \cdots$$

$$|E_n| = \frac{E_{mn}}{\sqrt{2}}$$

$$|I_n| = \frac{I_{mn}}{\sqrt{2}}$$

有効電力(平均電力)は周波数が等しい電圧と電流との間の有効電力の総和

皮相電力

$$P_a = |E||I|$$

$$= \sqrt{(E_0^2 + |E_1|^2 + |E_2|^2 + \cdots) \cdot (I_0^2 + |I_1|^2 + |I_2|^2 + \cdots)}$$

力率

$$\cos\varphi = \frac{\text{有効電力}}{\text{皮相電力}} = \frac{P}{P_a}$$

$$= \frac{E_0 I_0 + |E_1||I_1|\cos\varphi_1 + |E_2||I_2|\cos\varphi_2 + \cdots}{\sqrt{(E_0^2 + |E_1|^2 + |E_2|^2 + \cdots)\cdot(I_0^2 + |I_1|^2 + |I_2|^2 + \cdots)}}$$

[例題31.2]

$$v(t) = 4\sqrt{2}\sin(t - 30^\circ) + 2\sqrt{2}\sin(2t + 30^\circ) + 2\sqrt{2}\sin(3t - 30^\circ) + \sqrt{2}\sin(4t - 60^\circ)$$

$$i(t) = 5\sqrt{2}\sin(t - 30^\circ) + 3\sqrt{2}\sin(2t + 30^\circ) + \sqrt{2}\sin(3t + 30^\circ) + \sqrt{2}\sin(4t + 30^\circ)$$

実効値

$$V = \sqrt{\left(\frac{4\sqrt{2}}{\sqrt{2}}\right)^2 + \left(\frac{2\sqrt{2}}{\sqrt{2}}\right)^2 + \left(\frac{2\sqrt{2}}{\sqrt{2}}\right)^2 + \left(\frac{\sqrt{2}}{\sqrt{2}}\right)^2}$$

$$= \sqrt{16 + 4 + 4 + 1}$$

$$= \sqrt{25} = 5 \ [\mathrm{V}]$$

実効値

$$I = \sqrt{\left(\frac{5\sqrt{2}}{\sqrt{2}}\right)^2 + \left(\frac{3\sqrt{2}}{\sqrt{2}}\right)^2 + \left(\frac{\sqrt{2}}{\sqrt{2}}\right)^2 + \left(\frac{\sqrt{2}}{\sqrt{2}}\right)^2}$$

$$= \sqrt{25 + 9 + 1 + 1}$$

$$= \sqrt{36} = 6 \ [\mathrm{A}]$$

$$v(t) = 4\sqrt{2}\sin(t - 30^\circ) + 2\sqrt{2}\sin(2t + 30^\circ) + 2\sqrt{2}\sin(3t - 30^\circ) + \sqrt{2}\sin(4t - 60^\circ)$$

$$i(t) = 5\sqrt{2}\sin(t - 30^\circ) + 3\sqrt{2}\sin(2t + 30^\circ) + \sqrt{2}\sin(3t + 30^\circ) + \sqrt{2}\sin(4t + 30^\circ)$$

有効電力

$$\begin{aligned} P &= 4 \times 5\cos(-30^\circ - (-30^\circ)) + 2 \times 3\cos(30^\circ - (30^\circ)) \\ &\qquad +2 \times 1\cos(-30^\circ - (30^\circ)) + 1 \times 1\cos(-60^\circ - (30^\circ)) \\ &= 4 \times 5\cos(0^\circ) + 2 \times 3\cos(0^\circ) + 2 \times 1\cos(-60^\circ) + 1 \times 1\cos(-90^\circ) \\ &= 4 \times 5\cos(0^\circ) + 2 \times 3\cos(0^\circ) + 2 \times 1\cos(60^\circ) + 1 \times 1\cos(90^\circ) \\ &= 4 \times 5 + 2 \times 3 + 2 \times 1 \times \frac{1}{2} + 1 \times 1 \times 0 \\ &= 20 + 6 + 1 \\ &= 27\ [\mathrm{W}] \end{aligned}$$

$$\cos(-\theta) = \cos(\theta)$$

力率

$$\cos\varphi = \frac{P}{V \cdot I} = \frac{27}{5 \times 6} = 0.9$$

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