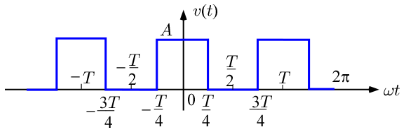
[問題30.1](1)

$\omega = \frac{2\pi}{T}$



$$f(t) = \sum_{n=1}^{\infty} a_n \sin n\omega t + b_0 + \sum_{n=1}^{\infty} b_n \cos n\omega t$$

偶関数より $a_n = 0$

b_0 を求める

(解法1)

$$b_0 = \frac{1}{T}\int_0^T v(t)\ dt = \frac{1}{T}\left(\int_0^{\frac{T}{4}} A\ dt + \int_{\frac{3T}{4}}^{T} A\ dt\right)$$

(解法2)半周期を2倍

$$b_0 = \frac{2}{T}\int_0^{\frac{T}{2}} v(t)\ dt = \frac{2}{T}\int_0^{\frac{T}{4}} A$$

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(解法3)周期を $-\frac{T}{4} \sim \frac{3T}{4}$ と考える

$$b_0 = \frac{1}{T}\int_{-\frac{T}{4}}^{\frac{3T}{4}} v(t)\ dt = \frac{1}{T}\int_{-\frac{T}{4}}^{\frac{T}{4}} A\ dt$$

(解法1)で解く

$$b_0 = \frac{A}{T}\left([t]_0^{\frac{T}{4}} + [t]_{\frac{3T}{4}}^{T}\right) = \frac{A}{T}\left(\frac{T}{4} + \frac{T}{4}\right) = \frac{A}{2}$$

(解法2)で解く

$$b_0 = \frac{2A}{T}[t]_0^{\frac{T}{4}} = \frac{2A}{T}\frac{T}{4} = \frac{A}{2}$$

(解法3)で解く

$$b_0 = \frac{A}{T}[t]_{-\frac{T}{4}}^{\frac{T}{4}} = \frac{A}{T}\left(\frac{T}{4} + \frac{T}{4}\right) = \frac{A}{2}$$

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b_n を求める

(解法1)で解く

$$b_n = \frac{2}{T}\int_0^T v(t)\cos n\omega t\ dt$$
$$= \frac{2}{T}\left(\int_0^{\frac{T}{4}} A\cos n\omega t\ dt + \int_{\frac{3T}{4}}^{T} A\cos n\omega t\ dt\right)$$
$$= \frac{2A}{T}\left(\left[\frac{1}{n\omega}\sin n\omega t\right]_0^{\frac{T}{4}} + \left[\frac{1}{n\omega}\sin n\omega t\right]_{\frac{3T}{4}}^{T}\right)$$
$$= \frac{2A}{n\omega T}\left(\sin n\omega\frac{T}{4} \underbrace{- \sin 0}_{=0} + \sin n\omega T - \sin n\omega\frac{3T}{4}\right)$$

$\omega T = 2\pi$ より

$$b_n = \frac{2A}{2\pi n}\left(\sin\frac{2\pi n}{4} \underbrace{+ \sin 2\pi n}_{=0} - \sin\frac{6\pi n}{4}\right)$$
$$= \frac{A}{\pi n}\left(\sin\frac{\pi n}{2} \underbrace{- \sin\frac{3\pi n}{2}}_{= \sin\frac{\pi n}{2}}\right) = \frac{2A}{\pi n}\sin\frac{\pi n}{2}$$

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$$b_1 = \frac{2A}{\pi}\sin\frac{\pi}{2} = \frac{2A}{\pi}$$
$$b_2 = \frac{2A}{2\pi}\sin\frac{2\pi}{2} = 0$$
$$b_3 = \frac{2A}{3\pi}\sin\frac{3\pi}{2} = -\frac{2A}{3\pi}$$

よって

$$v(t) = \frac{A}{2} + \frac{2A}{\pi}\left(1\cdot\cos\omega t - \frac{1}{3}\cos 3\omega t + \frac{1}{5}\cos 5\omega t + \cdots\right)$$

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(解法2)で解く *(解法1)との違いを示す

$$b_n = \frac{2}{T}\int_0^T v(t)\cos n\omega t\ dt$$
$$= \frac{\overset{4}{\cancel{2}}}{T}\left(\int_0^{\frac{T}{4}} A\cos n\omega t\ dt \cancel{+ \int_{\frac{3T}{4}}^{T} A\cos n\omega t\ dt}\right)$$
$$= \frac{\overset{4}{\cancel{2}}A}{T}\left(\left[\frac{1}{n\omega}\sin n\omega t\right]_0^{\frac{T}{4}} \cancel{+ \left[\frac{1}{n\omega}\sin n\omega t\right]_{\frac{3T}{4}}^{T}}\right)$$
$$= \frac{\overset{4}{\cancel{2}}A}{n\omega T}\left(\sin n\omega\frac{T}{4} \underbrace{- \sin 0}_{=0} + \cancel{\sin n\omega T - \sin n\omega\frac{3T}{4}}\right)$$

$\omega T = 2\pi$ より

$$b_n = \frac{\overset{4}{\cancel{2}}A}{2\pi n}\left(\sin\frac{2\pi n}{4} \cancel{+ \sin 2\pi n - \sin\frac{6\pi n}{4}}\right)$$
$$= \frac{\overset{2}{\cancel{A}}}{\pi n}\left(\sin\frac{\pi n}{2} \cancel{- \sin\frac{3\pi n}{2}}\right) = \frac{2A}{\pi n}\sin\frac{\pi n}{2}$$

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(解法3)で解く *(解法1)との違いを示す

$$b_n = \frac{2}{T}\int_0^T v(t)\cos n\omega t\ dt$$
$$= \frac{2}{T}\left(\int_{\cancel{0}\,-\frac{T}{4}}^{\frac{T}{4}} A\cos n\omega t\ dt \cancel{+ \int_{\frac{3T}{4}}^{T} A\cos n\omega t\ dt}\right)$$
$$= \frac{2A}{T}\left(\left[\frac{1}{n\omega}\sin n\omega t\right]_{\cancel{0}\,-\frac{T}{4}}^{\frac{T}{4}} \cancel{+ \left[\frac{1}{n\omega}\sin n\omega t\right]_{\frac{3T}{4}}^{T}}\right)$$
$$= \frac{2A}{n\omega T}\left(\sin n\omega\frac{T}{4} \underbrace{\cancel{- \sin 0 + \sin n\omega T - \sin n\omega\frac{3T}{4}}}_{\underbrace{- \sin\left(-n\omega\frac{T}{4}\right)}_{= + \sin n\omega\frac{T}{4}}}\right)$$

$\omega T = 2\pi$ より

$$b_n = \frac{\overset{4}{\cancel{2}}A}{2\pi n}\left(\sin\frac{2\pi n}{4} \cancel{+ \sin 2\pi n - \sin\frac{6\pi n}{4}}\right)$$
$$= \frac{\overset{2}{\cancel{A}}}{\pi n}\left(\sin\frac{\pi n}{2} \cancel{- \sin\frac{3\pi n}{2}}\right) = \frac{2A}{\pi n}\sin\frac{\pi n}{2}$$

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