第 24 章：非正弦波交流

24.4 ひずみ波の実効値

24.5 ひずみ率

キーワード：ひずみ波の実効値，ひずみ率

学習目標：ひずみ波の実効値，ひずみ率を計算できる。
ひずみ波の交流電力を求めることができる。

6 ひずみ波

6.4 ひずみ波の実効値

$$i(t) = I_0 + I_{m1}\sin(\omega t + \theta_1) + I_{m2}\sin(2\omega t + \theta_2) + \cdots$$

$$|I| = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt} \qquad \omega = \frac{2\pi}{T}$$

$$|I| = \sqrt{I_0^2 + |I_1|^2 + |I_2|^2 + \cdots}$$

証明

$$\begin{aligned} i(t)^2 &= (I_0 + I_{m1}\sin(\omega t + \theta_1) + I_{m2}\sin(2\omega t + \theta_2) + \cdots) \\ &\qquad \times (I_0 + I_{m1}\sin(\omega t + \theta_1) + I_{m2}\sin(2\omega t + \theta_2) + \cdots) \\ &= (I_0^2 + I_{m1}^2\sin^2(\omega t + \theta_1) + I_{m2}^2\sin^2(2\omega t + \theta_2) + \cdots) \\ &\quad + 2I_0(I_{m1}\sin(\omega t + \theta_1) + I_{m2}\sin(2\omega t + \theta_2) + \cdots) \\ &\quad + 2I_{m1}\sin(\omega t + \theta_1)(I_{m2}\sin(2\omega t + \theta_1) + \cdots) \end{aligned}$$

$$\cos 2\omega t = \cos^2 \omega t - \sin^2 \omega t$$
$$= 1 - 2\sin^2 \omega t$$
$$\Rightarrow \sin^2 \omega t = \frac{1 - \cos 2\omega t}{2}$$

$$\frac{1}{T}\int_0^T I_0^2 \; dt = I_0^2$$

$$\frac{1}{T}\int_0^T I_{mn}^2 \sin^2(n\omega t + \theta_n) \; dt$$
$$= \frac{I_{mn}^2}{2T}\int_0^T (1 - \cos 2(n\omega t + \theta_n)) \; dt = \left(\frac{I_{mn}}{\sqrt{2}}\right)^2 = |I_n|^2$$

$$\frac{1}{T}\int_0^T I_0 I_{mn} \sin(n\omega t + \theta_n) \; dt = 0$$

sin の積分は 0

$$\frac{1}{T}\int_0^T I_{mh} I_{mk} \sin(h\omega t + \theta_h)\sin(k\omega t + \theta_k) \; dt = 0 \qquad (h \neq k)$$

周波数が異なる sin 同士の積分は 0

$$\frac{1}{T}\int_0^T I_{mh}I_{mk}\sin(h\omega t+\theta_h)\sin(k\omega t+\theta_k)\ dt$$

$$= -\frac{I_{mh}I_{mk}}{2T}\int_0^T \cos((h+k)\omega t+(\theta_h+\theta_k)) - \cos((h-k)\omega t+(\theta_h-\theta_k))\ dt$$

$$= -\frac{I_{mh}I_{mk}}{2T}\left\{\frac{1}{h+k}\left[\sin((h+k)\omega t+(\theta_h+\theta_k))\right]_0^T - \frac{1}{h+k}\left[\sin((h-k)\omega t+(\theta_h-\theta_k))\right]_0^T\right\}$$

$$= -\frac{I_{mh}I_{mk}}{2T}\left[\frac{1}{h+k}\{\sin((h+k)\underbrace{\omega T}_{=2\pi}+(\theta_h+\theta_k)) - \sin(\theta_h+\theta_k)\} - \frac{1}{h+k}\{\sin((h-k)\underbrace{\omega T}_{=2\pi}+(\theta_h-\theta_k)) - \sin(\theta_h-\theta_k)\}\right]$$

$$= -\frac{I_{mh}I_{mk}}{2T}\left[\frac{1}{h+k}\{\underbrace{\sin((h+k)2\pi+(\theta_h+\theta_k))}_{=\sin(\theta_h+\theta_k)} - \sin(\theta_h+\theta_k)\} - \frac{1}{h+k}\{\underbrace{\sin((h-k)2\pi+(\theta_h-\theta_k))}_{=\sin(\theta_h+\theta_k)} - \sin(\theta_h-\theta_k)\}\right]$$

$$= 0$$

$$\frac{1}{T}\int_0^T i(t)^2\,dt = \frac{1}{T}\int_0^T \left(I_0^2 + I_{m1}^2\sin^2(\omega t+\theta_1) + I_{m2}^2\sin^2(2\omega t+\theta_2) + \cdots\right)$$

$$+2I_0\left(I_{m1}\sin(\omega t+\theta_1) + I_{m2}\sin(2\omega t+\theta_2) + \cdots\right)$$

$$+2I_{m1}\sin(\omega t+\theta_1)\left(I_{m2}\sin(2\omega t+\theta_1) + \cdots\right)$$

$$= I_0^2 + \frac{I_{m1}^2}{2} + \frac{I_{m2}^2}{2} + \cdots$$

$$= I_0^2 + \underbrace{\left(\frac{I_{m1}}{\sqrt{2}}\right)^2}_{=I_1^2} + \underbrace{\left(\frac{I_{m2}}{\sqrt{2}}\right)^2}_{=I_2^2} + \cdots$$

よって

$$|I| = \sqrt{I_0^2 + |I_1|^2 + |I_2|^2 + \cdots}$$

[例題 31.2]

$$v(t) = \underline{4\sqrt{2}}\sin(t - 30^\circ) + \underline{2\sqrt{2}}\sin(2t + 30^\circ)$$

$= \sqrt{2}|V_1|$ 　　　　 $= \sqrt{2}|V_2|$

$$+\underline{2\sqrt{2}}\sin(3t - 30^\circ) \underline{+ \sqrt{2}}\sin(4t - 60^\circ)$$

$= \sqrt{2}|V_3|$ 　　　　 $= \sqrt{2}|V_4|$

$$|V_1| = 4$$

$$|V_2| = 2$$

$$|V_3| = 2$$

$$|V_4| = 1$$

よって

$$|V| = \sqrt{|V_1|^2 + |V_2|^2 + |V_3|^2 + |V_4|^2}$$

$$= \sqrt{4^2 + 2^2 + 2^2 + 1^2} = 5$$

24 非正弦波交流

24.5 波形率・波高率・ひずみ率

$$i(t) = I_0 + I_{m1}\sin(\omega t - \theta_1) + I_{m2}\sin(2\omega t - \theta_2) + \cdots$$

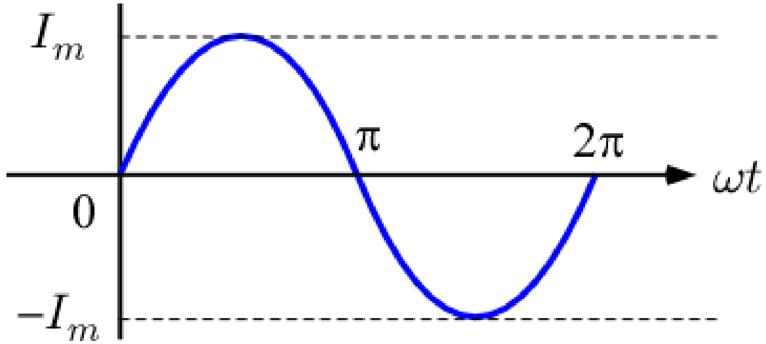
$$I_1 = \frac{I_{m1}}{\sqrt{2}}, \quad I_2 = \frac{I_{m2}}{\sqrt{2}}$$

$$\text{波形率} = \frac{\text{実効値}}{\text{平均値}} = \frac{|I|}{I_a} = \frac{\sqrt{\frac{1}{T}\int_0^T i^2\ dt}}{\frac{1}{T}\int_0^T |i|\ dt} = \frac{\sqrt{I_0^2 + I_1^2 + \cdots}}{\frac{1}{T}\int_0^T |i|\ dt}$$

$$\text{波高率} = \frac{\text{最大値}}{\text{実効値}} = \frac{I_m}{|I|} = \frac{I_m}{\sqrt{\frac{1}{T}\int_0^T i^2\ dt}} = \frac{I_m}{\sqrt{I_0^2 + I_1^2 + \cdots}}$$

$$\text{ひずみ率} = \frac{\text{全高調波の実効値}}{\text{基本波の実効値}} = \frac{\sqrt{|I_2|^2 + |I_3|^2 + \cdots}}{|I_1|}$$

正弦波 $i(t) = I_m \sin \omega t$



$$波形率 = \frac{\frac{I_m}{\sqrt{2}}}{\frac{2}{\pi} I_m} = \frac{\pi}{2\sqrt{2}} = 1.11$$

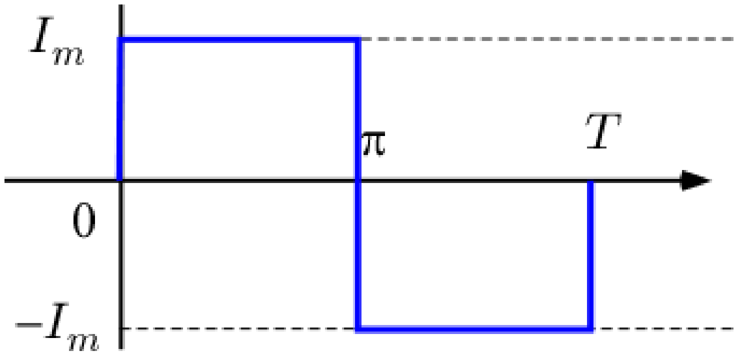
$$波高率 = \frac{I_m}{\frac{I_m}{\sqrt{2}}} = \sqrt{2} = 1.41$$

$$ひずみ率 = \frac{0}{\frac{I_m}{\sqrt{2}}} = 0$$

$2\omega t,\ 3\omega t, \cdots$ はない

方形波

$$i(t) = \begin{cases} I_m & \left(0 < t < \frac{T}{2}\right) \\ -I_m & \left(\frac{T}{2} < t < T\right) \end{cases}$$



$$\text{波形率} = \frac{I_m}{I_m} = 1$$

$$\text{波高率} = \frac{I_m}{I_m} = 1$$

$$i(t) = \frac{8}{T} I_m \left(\sin t + \frac{1}{3}\sin 3t + \frac{1}{5}\sin 5t + \cdots \right)$$

$$\text{ひずみ率} = \frac{\sqrt{\left(\frac{1}{3\sqrt{2}}\right)^2 + \left(\frac{1}{5\sqrt{2}}\right)^2 + \cdots}}{\frac{1}{\sqrt{2}}} = \frac{\sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{5}\right)^2 + \cdots}}{1}$$

$$= \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{5}\right)^2 + \cdots}$$

[例題 31.2]

$$v(t) = \underline{4\sqrt{2}}\sin(t-30^\circ) + \underline{2\sqrt{2}}\sin(2t+30^\circ)$$

$$= \sqrt{2}|V_1| \qquad\qquad = \sqrt{2}|V_2|$$

$$+\underline{2\sqrt{2}}\sin(3t-30^\circ) + \underline{\sqrt{2}}\sin(4t-60^\circ)$$

$$= \sqrt{2}|V_3| \qquad\qquad = \sqrt{2}|V_4|$$

ひずみ率

$$k = \frac{\sqrt{|V_2|^2 + |V_3|^2 + |V_4|^2}}{|V_1|}$$

$$= \frac{\sqrt{2^2 + 2^2 + 1^2}}{4} = \frac{3}{4}$$

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